# Analysis of coupling phenomena in a TEM concentric array of applicators radiating into a layered biological tissue model

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## I. Introduction

The principle idea of using positive interference arising from various sources to create focusing of electromagnetic waves inside biological tissues has been extensively employed, based on the use of continuous wave signals. A major advantage of coherent radiating multi-element heating systems consisting of waveguides or multiple dipole antennas is that they provide the possibility of controlling the electromagnetic energy deposited in the tissue and optimizing the heating for any given body model by phase and amplitude adjustment [1]-[2]. However, a significant limitation of this idea has been related to the side effects created by the coupling phenomena between array source elements. The experience with annular phased array systems [3] consisting of dielectrically loaded horn antennas was that a significant interaction between radiators is normally observed, while phantom measurements with the four-waveguide system [4], indicated that significant coupling between elements occurs via the radiating apertures. Furthermore, theoretical studies have shown that strong coupling phenomena are observed in case of near field applications where a concentric array of $TE_{10}$ waveguides is used [5].

In the present work, the coupling between the waveguide applicators of a TEM concentric array, radiating into a three-layer cylindrical body model of circular cross section is treated semianalytically, by using the scattering matrix ($\bar{\underline{S}}$) notation. In the following analysis an $\exp(+j\omega t)$ time dependence is assumed for the field quantities and it is suppressed throughout the analysis.

## II. Mathematical formulation and analysis

The system examined in this paper consists of an arbitrary number ($L$) of identical parallel-plate waveguide applicators, symmetrically placed at the periphery of a three-layer cylindrical lossy model of circular cross section. The three layers can be used to simulate different biological media, such as skin, bone and brain tissues. The dielectric properties of the layers are denoted with

0-7803-4478-2/98/$10.00 

the corresponding relative complex permittivities $\varepsilon_1, \varepsilon_2, \varepsilon_3$, while the magnetic permeabilities are assumed to be $\mu_1 = \mu_2 = \mu_3 = \mu_0$. The free-space wavenumber is $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$, where $\varepsilon_0$ and $\mu_0$ are the free-space permittivity and permeability, respectively. The applicators are filled with a dielectric material of relative permittivity $\varepsilon_w$ and relative permeability $\mu_w$ and have an aperture size of $b$ circulating around the cylindrical body's surface, while their infinite dimension is parallel to the cylindrical body's axis. It is assumed that the radiating apertures are conforming to the cylindrical body's surface and are separated by perfectly conducting flanges.

The fields inside the tissue layers ($i$=1,2,3) are expressed in terms of cylindrical vector wave functions [6],

$$\underline{E}_i(\underline{r}) = \sum_{m=-\infty}^{m=+\infty} (a_{im} \underline{M}^{(1)}_{m,k}(\underline{r}, k_i) + b_{im} \underline{N}^{(1)}_{m,k}(\underline{r}, k_i) + a'_{im} \underline{M}^{(1)}_{m,k}(\underline{r}, k_i) + b'_{im} \underline{N}^{(1)}_{m,k}(\underline{r}, k_i)) \qquad (1)$$

where $k_i = k_0\sqrt{\varepsilon_i}$ and $a_{im}, b_{im}, a'_{im}, b'_{im}$ are to be determined.

Next, the fields inside the waveguide applicators are described, by using the waveguides normal modes. If the operating frequency is lower than the $TM_1$ cutoff frequency, only a single mode, the TEM mode, with a zero cutoff frequency, will be propagating in the waveguides. Thus, the fields inside each waveguide applicator are expressed as the superposition of the incident TEM mode and an infinite number of all the reflected TEM and TM modes, since TE modes are not excited due to the geometry. Thus, the transverse electric field in the $\ell$th applicator ($\ell = 1,2,\ldots,L$) can be written, with respect to the local Cartesian coordinates of the $\ell$th waveguide [6], as follows:

$$\underline{E}^w_{\ell,t} = A_\ell \underline{e}^{EM}_t(y_\ell)e^{-jkz_\ell} + A'_\ell \underline{e}^{EM}_t(y_\ell)e^{jkz_\ell} + \sum_{m=1}^{\infty}[\frac{j\lambda_m}{v_m} B'_{\ell,m} \underline{e}^M_{m,t}(y_\ell)e^{j\lambda_m z_\ell}] \qquad (2)$$

where the subscript $t$ is used to denote the transverse field components, $A_\ell$ is the complex amplitude of the excited TEM mode in the $\ell$th waveguide and $A'_\ell, B'_{\ell,m}$ are the complex amplitudes of the reflected TEM, $m$th order TM modes, respectively, in the $\ell$th waveguide and $k$, $\lambda_m$ are the corresponding propagation constants.

By satisfying the continuity of the tangential electric and magnetic field components on the interfaces between different tissue layers and on the contact surface between cylindrical lossy model and radiating apertures, the following system of coupled integral equations is obtained, in terms of an unknown transverse electric field $\underline{E}_a$ on the waveguide apertures,

$$\sum_{q=1}^{L} \int_{\Gamma_q} \overline{\underline{K}}_{\ell q}(y/y')\underline{E}_a(y')dy' = 2A_\ell(\frac{k}{\omega\mu_0\mu_w})\underline{h}^{EM}_t(y), \ell = 1,2,\ldots,L/q = 1,2,\ldots,L \qquad (3)$$

where $\underline{h}_t^{EM}$ is the incident TEM mode transverse magnetic field on the aperture of the $\ell$th waveguide, and the kernel matrices $\overline{\underline{K}}_{\ell q}(y/y')$, $\ell = 1,\dots,L / q = 1,\dots,L$ indicate the effect of coupling from the $q$th aperture $(y') \in \Gamma_q$ to the $\ell$th aperture $(y) \in \Gamma_\ell$. In order to determine the electric field on the waveguide apertures, the system of integral equations (3) is solved. To this end a Galerkin's technique is adopted by expanding the unknown transverse electric field on each aperture $\underline{E}_{q,a}$ into wavequide normal modes,

$$\underline{E}_{q,a} = \sum_{m=1}^{\infty}(g_q \underline{e}_q^{EM} + f_{q,m}\underline{e}_{m,t}^{M}), \qquad q = 1,2,\dots,L \tag{4}$$

By substituting (4) into the system of coupled integral equations (3), and making use of the waveguide modes orthogonality [6], the later is converted into an infinite system of linear equations. By solving this system of linear equations, the expansion coefficients $g_{q,n}$ and $f_{q,n}$ of the field on each waveguide aperture are computed and then the scattering matrix coefficients can be easily obtained.

### III. Numerical Results

The effects of coupling between the elements of a TEM concentric array are studied, by using the scattering matrix notation involving self reflection and mutual coupling coefficients. By using the analysis presented in the previous sections, the performance of a eight element concentric array symmetrically placed at the periphery of a two-layer (bone-brain) biological tissue model, 16 cm in diameter, surrounded by a 2 cm thick lossless dielectric layer, is analyzed in detail. The applicators aperture size is defined by $b$=8 cm.

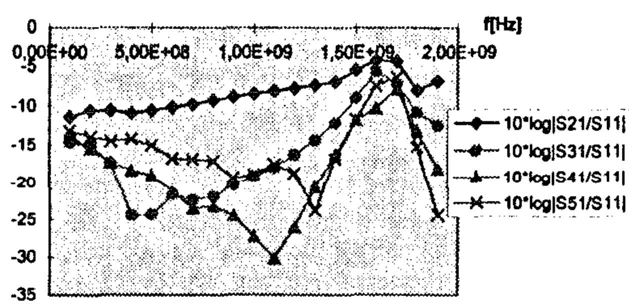


Fig.1: The ratio $|S_{\ell 1} / S_{11}|$ for $\ell = 2,3,4,5$, in dB, for a eight-element TEM concentric array of applicators, symmetrically placed at the periphery of a three-layer cylindrical lossy model, .20 cm in diameter.

The coupling coefficients between system elements may be readily computed by exciting one element and computing the amplitude and phase of the TEM mode coefficients coupled to the other waveguide applicators (mutual coupling coefficients) and the coefficient of the reflected TEM mode on the same aperture (self reflection coefficient). The ratio of the induced TEM mode coefficient at $\ell$th element ($\ell = 1,2,\ldots,8$) to the excitation TEM coefficient at $q$th element ($q=1,2,\ldots,8$) gives the amplitude and the phase of the coupling coefficient $S_{\ell q}$. In order to analyze the strength of the coupling phenomena, the ratios between the mutual coupling to self reflection coefficients are calculated at the operation range ($f < f_1$, $f_1$ being the cutoff frequency of the $TM_1$ mode) of the used TEM waveguides. In Fig.1, the ratio $|S_{\ell 1} / S_{11}|$ for $\ell = 2,3,4,5$, is presented in dB. It can easily be observed that the coupling between neighbouring applicators is stronger compared to distant applicators, while stronger coupling phenomena (`-5 dB) are observed at 1.6 GHz.

## VI. Conclusions

A semianalytical solution has been presented for the power coupling between the waveguide applicators of a TEM concentric array radiating into a three-layered cylindrical tissue model of circular cross section. Scattering parameters indicating the effect of coupling via the radiating apertures of a eight element TEM concentric array have been computed and presented at the operation range of the system.

## References


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