Comments on “A Modified Reachability Tree Approach to Analysis of Unbounded Petri Nets”

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Abstract

The above paper introduced the construction of a Modified Reachability Tree (MRT) for (unbounded) Petri nets and its application to reachability, liveness and deadlock analysis. This note shows via a counterexample that some of the MRT properties claimed in the above paper are incorrect.

In [1], a Modified Reachability Tree (MRT) approach was developed to extend the capability of Karp-Miller’s Finite Reachability Trees (FRTs) [2] in terms of solving reachability, liveness and deadlock problems in Petri nets. In [1], Theorem 3 states the following:

**Theorem 3 (Deadlock):** A Petri net has deadlocks if and only if its modified reachability tree contains terminal nodes and/or full conditional nodes.

![Figure 1: A Petri Net and its MRT.](image)

The Petri net shown in Fig.1(a) shows that the theorem is not correct. Following the MRT Generation Algorithm in [1], the MRT of the Petri net in Fig.1(a) can be obtained as shown in Fig.1(b). According to

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the theorem stated above, this Petri net has deadlocks because its MRT contains a full conditional node, namely $\mu_{full} = (0,1,\omega_0)$. However, this net is clearly deadlock-free: according to the analysis shown in Fig. 2, there does not exist terminal markings in the reachability set (note that the structure in the box with $n \geq 1$ is a “basic unit” of the Petri net behavior. The reason for the failure of Theorem 3 of [1] in this example is that the instance $(0,1,0)$ of the full conditional node $\mu_{full}$, which is a deadlock marking, is unreachable. In other words, the set of markings represented by the nodes of the MRT cover the set of reachable markings but it is not necessarily equal to that set, which implies that Theorem 2 of [1] is also incorrect. One way to remedy the problems is to remove the “if” part in Theorem 3 of [1] and also modify Theorem 2 appropriately. Another possibility is to modify the MRT construction procedure in a way that eliminates unreachable markings. The latter approach, however, appears to require considerable additional effort.

References
