Dynamical models for fault detection in squirrel cage induction motors

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Abstract: Induction motors are the dominant components in industrial processes involving electromechanical energy conversion. Safety, reliability and efficiency are major concerns in modern induction motor applications. Since detecting faults on time could avoid costly unscheduled shutdowns, in recent years there has been an increased interest in induction motor fault detection and diagnosis.

In this paper, we propose monitoring schemes to solve fault detection problems of induction motors. We begin with a monitoring scheme to detect detuning operation in Indirect Field Oriented Control (IFOC) driven induction motors. Secondly, we present a monitoring scheme to detect broken rotor bars on IFOC-driven induction motors. The proposed monitoring scheme does not rely on spectral methods; instead, it monitors a carefully selected induction motor state, using an online observer. The key to fault detection is the development of a simplified dynamic model of a squirrel cage induction motor with broken rotor bars. Numerical simulations validate both monitoring schemes.

Keywords: fault detection; detuning; broken rotor bars; induction motor; differential geometry.

1 Introduction

Traditionally, fault detection problems in energy processing systems have been addressed in a model-free framework. A carefully selected signal is monitored in time or frequency domain in order to track deviations from its expected value. Only more elaborate monitoring schemes include the identification of false alarms. Since false alarms could cause costly shutdowns, this state of affairs is not completely satisfactory. Improvements that have been considered, mostly in the research literature, include the use of dynamical models. Two main stumbling blocks remain on this approach: one is the need to tailor the level of detail for component models (so as to keep the overall model tractable) and the other is the need to discriminate the sources of possible misclassification in a systematic fashion.

This paper presents a review of model-based fault detection via analytic redundancy to develop a comprehensive framework for fault detection in energy processing systems. This framework is then employed as a tool to design monitoring schemes for the induction motor, which is the dominant component in industrial processes involving electro-mechanical energy conversion. Given the main impediments to our approach, we introduce dynamic models with two important characteristics: simple enough to be tractable and detailed enough to capture the fault effects of interest. We place a special emphasis on the elimination of the effects of the sources of false alarms on the monitoring scheme.
The paper is organised as follows. Section 2 reviews model-based fault detection techniques. The review covers failure detection via analytic redundancy, starting from the pioneering work of Beard and Jones, and ending at the detection schemes based on differential geometric techniques. Section 3 proposes a monitoring scheme to detect detuned operation of Indirect Field Oriented Control (IFOC) driven induction motors. Section 4 presents a monitoring scheme to detect broken rotor bars in IFOC-driven squirrel cage induction motors, and is followed by the concluding remarks.

2 Model-based fault detection

Hardware redundancy appears to be the traditional engineering method to achieve fault detection in dynamic systems. Repeated sensors, actuators and system components are placed around the system to achieve fault detection. Such schemes often operate in a triplex or quadruplex configuration, and the redundant signals are compared for consistency. The simplicity of this approach accounts for its wide use. However, the simplicity is paid for in terms of the economic cost of implementation, and the required extra space to accommodate redundant components. Moreover, it is recognised in practice that similar components tend to fail at the same time of operation.

Since the early 1970s, new methods for fault detection have emerged to overcome the drawbacks of hardware redundancy, e.g., functional redundancy and analytic redundancy. In the functional redundancy approach, the redundant hardware elements are constructed from dissimilar components and the new outputs are compared with more sophisticated schemes. Analytical redundancy schemes are basically signal processing techniques, using state estimation, parameter estimation, statistical decision theory and logical and combinational operations. All signal processing techniques are practically implemented in electronic circuits and computers.

The basic function of a fault detection scheme is to produce an alarm when a fault occurs in the monitored system, and in a second stage to identify the failed component. Ideally, the alarm should be a binary signal announcing that a fault is affecting the system or that no fault is hampering the system. Another important characteristic for the alarm is the speed of detection. It is desirable to have a fault detection scheme that produces an alarm immediately after the occurrence of the fault. Finally, the most important characteristic of the alarm is the rate of false alarms, as false alarms make the performance of fault detection schemes deteriorate. Thus, a great effort in the design of fault detection schemes focuses on the problem of generation of alarms providing effective discrimination between different faults, system disturbances and modelling uncertainties.

2.1 Fault detection via analytic redundancy

Residuals, also denoted as alarms, are quantities expressing the difference between the actual plant outputs and those expected on the basis of the applied inputs and the mathematical model. They are obtained by exploiting dynamic or static relationships among sensor outputs and actuator inputs. An important characteristic for residuals is that they need to be robust with respect to the effect of nuisance faults, otherwise, nuisance faults will obscure the residual’s performance by acting as a source of false alarms.
The general procedure of Fault Detection, Isolation and Accommodation (FDIA) in dynamic systems with the aid of analytical redundancy consists of the following three steps (Patton et al., 1989):

1. generation of functions that carry information about the faults, so-called residuals
2. decision on the occurrence of a fault and localisation of the fault, so-called isolation
3. accommodation of the faulty process, and transition to normal operation.

This paper focuses attention on the generation of residuals using the analytic redundancy framework with priority on the robustness with respect to nuisance faults.

Robustness of the residual generator with respect to nuisance faults understood as a complete decoupling from the residual to nuisance faults is one of the most difficult problems in fault detection. To date, much of the work on the generation of residuals performed in the analytic redundancy framework observes two important tendencies: failure detection filters and parity relations. The similarities between these two approaches lead to the same residuals for linear systems and for some non-linear systems (Gertler, 2002).

2.2 Failure detection filters

Beard (1971) was the first to propose a fault detection filter, refined later by Jones (1973). In this filter, known as the Beard-Jones Detection (BJD) filter, the reachable subspaces of each fault are placed into invariant independent subspaces. Then, when a non-zero residual is detected, the fault is identified by projecting the residual onto the reachable subspaces of faults and comparing the projection with a threshold. Even though multiple faults can be detected with this filter, the approach is very restrictive as faults have to satisfy a mutual detectability condition (Massoumnia, 1986).

Further improvements to the BJD filter were suggested in (Massoumnia et al., 1989) with the Restricted Diagonal Detection (RDD) filter. In this approach, faults are divided into faults that need to be detected (so-called target faults) and nuisance faults (such as parameter uncertainties, changes in system parameters and noise). Nuisance faults are projected onto the unobservable space of residuals while maintaining observability of target faults; then, target faults are identified as in the BJD filter. When every fault is detected, BJD and RDD filters are equivalent.

The most recent version of the BJD filter, so-called Unknown Input (UI) observer approach, was proposed in (Patton et al., 1989) using the eigenstructure assignment and the Kronecker canonical form as design methods, and in (Massoumnia et al., 1989) using geometric techniques. In the UI observer approach, nuisance faults are projected onto the unobservable subspace so that residuals are influenced only by the target fault, in this way, simplifying the decision task. It should be pointed out that even though the use of UI observers in fault detection and isolation was first proposed in (Massoumnia et al., 1989; Patton et al., 1989), UI observers have received significant attention only after the pioneering work of Basille and Marro, presented for instance in (Basille and Marro, 2002). We consider the UI observer approach with geometric techniques.

In the non-linear setting, the residual generation problem using analytic redundancy has been addressed in (Hammouri et al., 1998) for state-affine systems and later in (De Persis and Isidori, 2001) for input-affine systems. In these works, residual generator construction is based, under mild additional assumptions, on the existence of an
unobservability subspace (distribution) leading to a subsystem unaffected by all fault signals but the fault of interest; then an asymptotic observer for such a subsystem, which in the non-linear case may not exist, yields the residual generator.

Consider the following non-linear system:

\[ \dot{x} = f(x) + g(x)u + l_m(x)m + l_t(x)m_t \]
\[ y = h(x) \]

where \( x \in \mathbb{R}^n \) is the state, \( y \in \mathbb{R}^4 \) is the measurable output, \( u \in \mathbb{R}^m \) is the input and \( m, m_t \) are arbitrary unknown functions of time, representing the target failure modes and the nuisance failure modes, respectively. During fault-free operation, \( m \) and \( m_t \) are equal to zero. The columns of \( f(x), g(x), l_m(x), l_t(x) \) and \( h(x) \) are smooth vector fields with \( l_m(x) \) and \( l_t(x) \) denoting the actuator failure signatures.

The residual generation problem may be stated as follows.

Problem 1 Consider the non-linear system described by Equation (1). Design, if possible, a dynamic residual generator with state \( x_R \) of the form:

\[ \hat{x} = F(\hat{x}, y) + E(\hat{x}, y) u \]
\[ r = M(\hat{x}, y) \]

where \( F(\hat{x}, y) \) and the columns of \( E(\hat{x}, y) \) are smooth vector fields and \( M(\hat{x}, y) \) is a smooth mapping, that takes \( y \) and \( u \) as inputs and generates the residual \( r \) with the following local properties:

1. When the target failure \( m_t \) is not present, the residual generator Equation (2) is asymptotically stable and \( r \) decays asymptotically to zero, that is, the transmission from the input \( u \) and the nuisance faults \( m_n \) to the residual is zero.
2. For a non-zero target fault, the residual is non-zero.

Condition 1 considers the stability of the residual generator and assures that the input signal \( u \) and the nuisance faults \( m_n \) do not affect the residual \( r \). Condition 2 guarantees that the target fault affects the residual.

In De Persis and Isidori (2001), a necessary condition for the existence of a solution to Problem 1 is given. This condition, under mild assumptions, leads to a subsystem driven only by the fault of interests. Thus, a solution to Problem 1 can be found provided such a subsystem admits an observer.

Specifically, assume that the minimal unobservability distribution of Equation (1), denoted by \( S^* \), containing the image of the nuisance fault signature \( L_n = \text{span}\{l_n(x)\} \), is locally non-singular. In De Persis and Isidori (2001), it is shown that if:

\[ S^* \cap \text{span}\{l_t(x)\} = \{0\} \]

then it is possible, under certain conditions, to find a state diffeomorphism and an output diffeomorphism:

\[ z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \Phi(x), \quad \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \Psi(y) \]
such that in the new coordinates, the system Equation (1) is described by equations of the form:

\[
\dot{z}_1 = \tilde{f}_1(z_1, z_2) + \tilde{g}_1(z_1, z_2) u + \tilde{l}_1(z) m_i,
\]

\[
\dot{z}_2 = \tilde{f}_2(z) + \tilde{g}_2(z) u + \tilde{l}_2(z) m_i + \tilde{i}_{c_2}(z) m_n,
\]

\[
\dot{z}_3 = \tilde{f}_3(z) + \tilde{g}_3(z) u + \tilde{l}_3(z) m_i + \tilde{i}_{c_3}(z) m_n,
\]

\[w_1 = \tilde{h}_1(z_1),\]

\[w_2 = z_2,\]

from where it is possible to extract a subsystem driven only by the target fault as:

\[
\dot{z}_1 = \tilde{f}_1(z_1, w_2) + \tilde{g}_1(z_1, w_2) u + \tilde{l}_{11}(z) m_i
\]

\[w_1 = \tilde{h}_1(z_1).\] (5)

Clearly, when it is possible to construct an observer for Equation (5), Problem 1 is solvable.

The computation of the minimal unobservability distribution \(S^*\) containing \(L_n\) can be computed as the last element of the following sequence (De Persis and Isidori, 2001):

\[
S_0 = W^* + \text{Ker}\{dh\},
\]

\[
S_i = W^* + \{f, S_{i-1} \cap \text{Ker}\{dh\}\}
\]

\[+ \{g, S_{i-1} \cap \text{Ker}\{dh\}\}, i = 1, ..., k,\] (6)

where \(k \leq n - 1\) is determined by the condition \(S_k = S_{k-1}\). Concerning \(W^*\), it is computed as the last element of the following sequence:

\[
W_0 = \overline{P},
\]

\[
W_i = \overline{W}_{i-1} + \{f, \overline{W}_{i-1} \cap \text{Ker}\{dh\}\}
\]

\[+ \{g, \overline{W}_{i-1} \cap \text{Ker}\{dh\}\}, i = 1, ..., k,\] (7)

with \(k \leq n - 1\) determined by the condition \(W_{i+1} = W_i\). In Equations (6) and (7), \(\ldots\) is the Lie product, \(\overline{X}\) denotes the involutive closure of \(X\), and \(P = \text{span}\{l_n(x)\}\).

3 Detuning detection and accommodation on IFOC driven induction motors

In most high-performance applications of electric drives (like speed and position servos), the control structure utilised is the so-called field-oriented (vector) control. It allows for (almost) decoupled control of torque and flux, and yields a very fast transient response. In the case of a well-tuned controller, the main performance limitations come from the current bandwidth of the drive. With modern power electronic switching devices (like IGBT) that bandwidth is well above a kHz, resulting in outstanding electro-mechanical response. In the case of a detuned operation, however, the performance can degrade substantially, both in transients (torque command following) and in steady state
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If the detuning is caused by a typically non-monitored process (like rotor time constant variation, or a slow degradation of the shaft position sensor information), it may go undetected for a while, and lead to drastic efficiency reduction, or even a hard fault. In this paper we propose model-based scheme to detect the detuning process in a general purpose field-oriented induction drive motor. It is based on differential geometric considerations, and it is immune to a number of transients that normally occur in a high-performance drive, like load torque variations.

3.1 Detuned operation of current-fed indirect field-oriented controlled induction motors

It is well known that mechanical commutation simplifies significantly the control task in DC motors. The action of the commutator is to reverse the direction of the armature winding currents as the coils pass the brush position so that the armature current distribution is fixed in space regardless of the rotor speed. Thus, the field flux produced by the stator and the Magneto-Motive Force (MMF) created by the current in the armature winding are maintained in a mutually perpendicular orientation independent of the rotor speed. The result of this orthogonality is that the field flux is practically unaffected by the armature current, so that, when the field flux is kept constant, the produced electro-mechanical torque is proportional to the armature current. Highly dynamic performance can be obtained using two linear control loops, one (slow) controlling the field flux and the other (fast) one controlling the armature current.

In induction motors field flux and armature MMF distributions are not orthogonal, rendering the analysis and control of these devices more complicated. However, the action of the commutator of a DC machine in holding a fixed, orthogonal spatial angle between the field flux and the armature MMF can be emulated in induction machines by orienting the stator current with respect to the rotor flux so as to attain practically independent controlled flux and torque. Such controllers are called field-oriented controllers and require independent control of both magnitude and phase of the AC quantities.

An understanding of the decoupled flux and torque control resulting from field orientation can be attained from the model of an induction machine in the fixed stator frame\(^1\) (Krause et al., 1995; Mohan, 2001; Novotny and Lipo, 1996):

\[
\sigma \frac{d}{dt} i_{as} = - \left( r_s + \frac{L_m}{L_s \tau_s} \right) i_{as} + \frac{L_m}{L_s \tau_s} \lambda_{ps} + \frac{L_m}{L_s \tau_s} \lambda_{sr} + \nu_{as}
\]

\[
\sigma \frac{d}{dt} i_{ps} = - \left( r_s + \frac{L_m}{L_s \tau_s} \right) i_{ps} + \frac{L_m}{L_s \tau_s} \lambda_{as} + \frac{L_m}{L_s \tau_s} \lambda_{pr} + \nu_{ps}
\]

\[
\dot{\lambda}_{ar} = - \frac{1}{\tau_r} \lambda_{ar} - \omega_r \lambda_{pr} + \frac{L_m}{\tau_s} i_{as}
\]

\[
\dot{\lambda}_{pr} = - \frac{1}{\tau_r} \lambda_{pr} + \omega_r \lambda_{ar} + \frac{L_m}{\tau_s} i_{ps}
\]

\[
\frac{2J}{P} \omega_r = \frac{P}{2} \frac{L_m}{L_s} (\lambda_{as} i_{ps} - \lambda_{ps} i_{as}) - \tau_L
\]
where $i_{ab}$ are the stator currents, $\lambda_{ab}$ are the rotor flux linkages, $v_{ab}$ are the stator voltages, $\omega_m = \frac{P}{2} \omega_e$ is the mechanical velocity, $P$ is the number of poles in the machine, $J$ is the rotational inertia, $\tau_L$ is the torque load, $r_s$, $r_r$ are the stator and rotor resistances, $L_s$, $L_r$, $L_m$ are the stator, rotor and mutual inductances, $L_m$.

The field orientation concept implies that the current components supplied to the machine should be oriented in phase (flux components) and in quadrature (torque component) to the rotor flux vector $\lambda_{ab}$. This can be accomplished by choosing $\omega_e$ to be the instantaneous speed of $\lambda_{ab}$ and locking the phase of the reference system to the direction of the magnetising flux $\lambda_m$, that is:

$$ e^{-j\omega_e t} \begin{bmatrix} \lambda_M \\ 0 \end{bmatrix} = e^{-j\omega_e} \begin{bmatrix} \lambda_{ar} \\ \lambda_{pe} \end{bmatrix} (9) $$

where:

$$ e^{j\omega_e t} = \begin{bmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{bmatrix}. $$

Assuming that the machine is supplied from a current regulated source, the stator equations can be omitted; the rotor dynamics, in terms of stator currents and rotor flux, in a rotor field-oriented frame is described by the following equations:

$$ \dot{\lambda}_m = \frac{1}{\tau_r} \lambda_m - \frac{L_m}{\tau_r} i_{sb} \quad (10) $$

$$ \omega_e = \omega_r + \frac{L_m}{\tau_r} \frac{i_{sb}}{\lambda_m} (11) $$

$$ \frac{2J}{P} \dot{\omega}_r = \frac{P}{2} \frac{L_m}{L_r} \lambda_m i_{qs} - \tau_L (12) $$

with $i_{sd} = e^{-j\theta_e} i_{sb}$.

Equations (10)–(12) describe the dynamic response of a field-oriented induction machine and are essentially parallel the DC machine dynamics. Equation (10) corresponds to the field circuit on a DC machine. Equation (11) defines what is commonly called the slip frequency $\omega_s = \omega_e - \omega_r$, which is inherently associated with the division of the input stator current into the desired flux and torque components. The electro-mechanical torque in Equation (12) shows the desired torque control property of providing a torque proportional to the torque command current $i_{qs}$.

The implementation of field orientation can be easily carried out, provided the position angle of the rotor flux $\theta_e$ is known. There are two basic approaches to determine $\theta_e$: direct schemes, which determine the angle from flux measurements, and indirect schemes, which measure the rotor velocity and utilise the slip frequency to compute the
angle of the rotor flux relative to the rotor. The indirect method uses the fact that a necessary condition to produce field orientation is to satisfy the slip relation. An indirect field-oriented controller is described by the following equations:

$$\dot{\lambda}_m = -\frac{1}{\hat{\tau}_r} \hat{\lambda}_m + \frac{L_m^*_m}{\hat{\tau}_r} \hat{i}_{dq}$$

$$\dot{\omega}_r = \omega_r + \frac{L_m^*_m}{\hat{\tau}_r} \frac{\hat{\lambda}_m}{\hat{\lambda}_m}$$

(13)

where $\hat{\omega}_r$ is the estimated synchronous velocity, $\hat{\lambda}_m$ is the estimated rotor flux linkage, $\hat{\tau}_r$ is the estimated rotor time constant and $\hat{i}_{dq}$ are the measured stator currents. The indirect field-oriented controller together with the current controller and the speed controller are shown in Figure 1. From Equations (10), (11) and (13) we observe that $\hat{\omega}_r = \omega_r$ and $\hat{\lambda}_m = \lambda_m$, provided all parameters are accurately known. As a result, it follows that $\theta_e = \hat{\theta}_e$ and the measured stator currents $\hat{i}_{dq}$ are equal to the actual stator currents $i_{dq}$. It is reasonable to assume that we have a good estimate of $L_m^*$; however, the rotor time constant is usually not exactly known as it changes because of motor heating, mismatch in manufacturing or other variations. A mismatch in the rotor time constant results in a loss of the correct field orientation, labelled as detuning of the controller. The main consequences of detuning are:

- The flux level is not properly maintained.
- The resulting steady state is not the commanded value.
- The torque response is degraded.
- The efficiency is degraded and the motor heating increases.

Figure 1  Indirect field-oriented control diagram
Next, we obtain a dynamic model that accounts for the detuning effect. Although the true field-oriented induction motor dynamics Equations (10)–(12) exist in the motor, its states cannot be measured directly. In fact, the only measurable outputs of the induction motor are the stator currents in the abc frame and the mechanical rotor speed $\omega_r$. Since it is assumed that the induction motor is wye connected, the currents $i_{abc}$ can be obtained from $i_{abcs}$ as follows:

\[
\begin{align*}
    i_a &= \frac{\sqrt{3}}{2} i_n, \\
    i_d &= \frac{1}{\sqrt{2}} (i_n - i_s).
\end{align*}
\]

As observed in Figure 2, the relation between currents $i_{dq}$ and currents $i_{dq'}$ is defined by:

\[
i_{dq'} = e^{-j \theta_e} i_{dq}
\]

Note now that in a detuned condition ($\theta_s \neq \hat{\theta}_s$), the measured stator currents are given as:

\[
\hat{i}_{dq'} = e^{-j \hat{\theta}_e} i_{dq'}
\]

thus, from Equations (15) and (16), we conclude that:

\[
i_{dq'} = e^{-j \theta_e} \hat{i}_{dq'}
\]

with $\hat{\theta}_e = \theta_e - \hat{\theta}_s$.

**Figure 2** Fixed frame $ab$ and rotating frames $dq$ and $\hat{dq}$.

Replacing Equation (17) into Equations (10)–(12) and re-ordering terms, we get an indirect field-oriented controlled induction motor model that includes detuning effects described by the following equations:
\[
\dot{\lambda}_m = -\frac{1}{\tau_r} \lambda_m + \frac{L_m}{\tau_r} \left[ \cos(\hat{\theta}_r) \dot{i}_{dq} + \sin(\hat{\theta}_r) \dot{i}_{dq} \right]
\]
\[
\dot{\theta}_r = \frac{L_m}{\tau_r} \cos(\hat{\theta}_r) \dot{i}_{dq} - \sin(\hat{\theta}_r) \dot{i}_{dq} - \frac{L_m}{\tau_r} \frac{\dot{i}_{dq}}{\lambda_m}
\]
\[
\dot{\lambda}_m = -\frac{1}{\dot{\tau}_r} \lambda_m + \frac{L_m}{\dot{\tau}_r} \dot{i}_{dq}
\]
\[
\frac{2J}{P} \omega_r = \frac{P}{2} \frac{L_m}{L_r} \lambda_m \left[ \cos(\hat{\theta}_r) \dot{i}_{dq} - \sin(\hat{\theta}_r) \dot{i}_{dq} \right] - \tau_e
\]

where \( \hat{\theta}_r = \omega_r - \hat{\omega}_r \). Note now that in steady state, that is when:
\[
\dot{\lambda}_m = \lambda_m = \omega_r = \dot{\omega}_r = 0,
\]
from Equation (18), we have that:
\[
\frac{1}{\tau_r} \cos(\hat{\theta}_r) \dot{i}_{dq} - \sin(\hat{\theta}_r) \dot{i}_{dq} = \frac{1}{\tau_r} \dot{i}_{dq} \quad \frac{1}{\tau_r} \cos(\hat{\theta}_r) \dot{i}_{dq} + \sin(\hat{\theta}_r) \dot{i}_{dq} = \frac{1}{\tau_r} \dot{i}_{dq}
\]
from which we obtain:
\[
\tan(\hat{\theta}_r) = \frac{\left( \frac{1}{\tau_r} \dot{i}_{dq} \right)}{\left( \frac{1}{\tau_r} \dot{i}_{dq} \right)}
\]
thus, \( \hat{\theta}_r = 0 \) provided \( \tau_r = \dot{\tau}_r \).

### 3.2 Detection of the detuned operation

Next, we design a residual generator to detect detuned indirect field-oriented controllers following the model-based-based fault detection method outlined in the previous section. We consider \( \hat{\theta}_r \) in Equation (18) as an externally generated signal and, for fault detection, we consider the following system:
\[
\dot{\lambda}_m = -\frac{1}{\tau_r} \lambda_m + \frac{L_m}{\tau_r} \left[ \cos(\hat{\theta}_r) \dot{i}_{dq} + \sin(\hat{\theta}_r) \dot{i}_{dq} \right]
\]
\[
\frac{2J}{P} \omega_r = \frac{P}{2} \frac{L_m}{L_r} \lambda_m \left[ \cos(\hat{\theta}_r) \dot{i}_{dq} - \sin(\hat{\theta}_r) \dot{i}_{dq} \right] - \tau_e
\]

Note now that to express the dynamics in Equation (20) in terms of the system Equation (1), we need to identify the target and nuisance faulty. Since the load torque \( \tau_L \) is also an unknown quantity that may vary over a wide range depending on the motor application, we consider it as a nuisance fault, that is:
\[ l_s = \begin{bmatrix} 0 \\ -\frac{P}{2J} \end{bmatrix}. \]

Note now that the information about detuning \( \tilde{\theta} \neq 0 \) is contained on the trigonometric functions in Equation (20), so by defining:

\[ m_i = \begin{bmatrix} \cos(\tilde{\theta}_r) - 1 \\ \sin(\tilde{\theta}_r) \end{bmatrix} \]

we have:

\[
\begin{align*}
\mathbf{i}_i &= \begin{bmatrix} \frac{L_m}{\tau_r} \dot{i}_{dq} - \frac{L_m}{\tau_r} \dot{i}_{dq} \\ \frac{p^2 L_m}{4J L_r} \lambda_m \dot{i}_{dq} - \frac{p^2 L_m}{4J L_r} \lambda_m \dot{i}_{dq} \end{bmatrix}, \\
\mathbf{f} &= \begin{bmatrix} \frac{1}{\tau_r} \lambda_m \\ 0 \end{bmatrix}, \\
\mathbf{g} &= \begin{bmatrix} \frac{L_m}{\tau_r} & 0 \\ 0 & \frac{p^2 L_m}{4J L_r} \lambda_m \end{bmatrix}
\end{align*}
\]

and \( \mathbf{u} = \begin{bmatrix} \dot{i}_{dq} \\ \dot{i}_{dq} \end{bmatrix}^T \).

As stated previously, the only measurable quantity in Equation (20) is the rotor speed \( \omega_r \), that is:

\[
y = \omega_r \quad (21)
\]

Straightforward computations show that for Equation (21) the minimal unobservability distribution is given as:

\[ \mathcal{S}_\omega = \text{span} \begin{bmatrix} 0 & 1 \\ \frac{1}{\tau_r} \\ -\frac{P}{2J} & 0 \end{bmatrix} \]

and condition Equation (3) is not satisfied.

Consider now the magnetising flux as the output of the system, that is:

\[
y_i = \lambda_m \quad (22)
\]

Analogous computations show that:
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\[ S_{\omega} = \text{span} \left\{ \begin{array}{c} 0 \\ \frac{p}{2J} \end{array} \right\} \]

and condition Equation (3) is satisfied.

By inspection, we note that in this case, the subsystem Equation (5), with \( z_i = \lambda_M \) and \( w_i = y_i \), reads as:

\[
\dot{\lambda}_M = -\frac{1}{r_r} \lambda_M + \frac{L_m}{r_r} \ddot{i}_d + \frac{L_m}{r_r} \ddot{i}_q + \frac{L_m}{r_r} \dot{i}_q m_i + \frac{L_m}{r_r} \dot{i}_d m_i
\]

Hence, we have a solution to Problem 1 by designing an observer for subsystem Equation (23). Note that in Equation (23) the rotor time constant is unknown. However, the residual generator:

\[
\dot{\lambda}_M = -\frac{1}{r_r} \lambda_M + \frac{L_m}{r_r} \ddot{i}_d - \Gamma (\ddot{\lambda}_M - \dot{\lambda}_M)
\]

\[ r = \ddot{\lambda}_M - \dot{\lambda}_M \]

where \( \Gamma > 0 \) solves Problem 1.

To verify that a solution to Problem 1 is given by Equation (24), note that the dynamics of the residual is described by the following equation:

\[
\dot{r} = -\Gamma r \left( \frac{1}{r_r} - \frac{1}{r_r} \right) (\dot{\lambda}_M - \ddot{\lambda}_M) - \frac{L_m}{r_r} \dot{i}_d m_i - \frac{L_m}{r_r} \dot{i}_q m_i
\]

The second right-hand term in Equation (25) is somewhat unexpected; however, we notice that this term will be zero provided \( m_i = 0 \) and \( m_q = 0 \). In fact, this term also represents the detuning problem expressed as the difference between the estimated rotor time constant and the actual rotor time constant. Hence, we have established that Equation (24) is a solution to Problem 1.

### 3.3 Estimation of the magnetising flux

Note that to compute the residual, we need to have access to the magnetising flux \( \lambda_M \), which is not typically available. However, as stated in Mijalkovic (2002), this flux can be computed as follows. Multiplying the first and the second equations in Equation (8) by \( i_{p_r} \) and \( -i_{a_r} \), respectively, and adding the resulting equations we have:

\[
\sigma \begin{bmatrix} i_{p_r} \\ \frac{d}{dt} i_{a_r} - i_{as} \end{bmatrix} = \omega \frac{L_m}{L_r} \left[ \lambda_{p_r} i_{p_r} + \lambda_{a_r} i_{a_r} \right] + \frac{L_m}{L_r} \left[ \lambda_{a_r} i_{p_r} + \lambda_{p_r} i_{a_r} \right] + v_{a_r} i_{p_r} - v_{p_r} i_{a_r}
\]

Simple computations show that:

\[
\lambda_{p_r} i_{p_r} + \lambda_{a_r} i_{a_r} = \lambda_M i_{sh}
\]

\[
\lambda_{a_r} i_{p_r} - \lambda_{p_r} i_{a_r} = \lambda_M i_{qs}
\]
thus, we have:

\[
\sigma \left[ i_{m} \frac{d}{dt} i_{m} - i_{a} \frac{d}{dt} i_{o} \right] = \frac{L_m}{L_r} \lambda_m \left[ \omega - \frac{1}{\tau_r} i_{o} \right] + \nu_{r}, i_{m} - \nu_{r}, i_{a}, \tag{26}
\]

Assuming that the rotor dynamics is in steady state, we have:

\[
\lambda_m = L_m i_{o}
\]
\[
\omega_r = \frac{\dot{i}_o}{\tau_r} \tag{27}
\]

Replacing Equation (27) into Equation (26), one has:

\[
\sigma \left[ i_{m} \frac{d}{dt} i_{m} - i_{a} \frac{d}{dt} i_{o} \right] = \frac{\dot{\omega}}{L_r} \lambda_m^2 + q \tag{28}
\]

where \( q = \nu_{r}, i_{m} - \nu_{r}, i_{a} \). Define now:

\[
\theta = \arctan \left( \frac{i_{m}}{i_{o}} \right) \tag{29}
\]

thus, Equation (28) can be written as:

\[
\dot{\theta} = \frac{\lambda_m}{\sigma \left( i_{m}^2 + i_{o}^2 \right)} \tag{30}
\]

Under the above assumption \( \lambda_m \) is a constant in Equation (30). In order to estimate \( \lambda_m \), we follow the general results presented in Karagiannis et al. (2003). Define the estimation error:

\[
z = \Lambda - \lambda_m^2 + \beta(\theta) \tag{31}
\]

thus, we have:

\[
\dot{z} = \dot{\Lambda} + \frac{\partial \beta(\theta)}{\partial \theta} \frac{\dot{\lambda}_m}{\sigma \left( i_{m}^2 + i_{o}^2 \right)} (\Lambda - z + \beta(\theta)) + q \tag{32}
\]

Defining:

\[
\beta(\theta) = K \sigma \theta
\]
\[
\dot{\Lambda} = -K \frac{\dot{i}_o}{i_{m}^2 + i_{o}^2} (\Lambda + K \sigma \theta) + q \tag{32}
\]

with \( K > 0 \), the estimation error dynamics is described by:

\[
\dot{z} = -\frac{K}{L_r \left( i_{m}^2 + i_{o}^2 \right)} z
\]
thus, $z$ converges exponentially to zero and:

$$
\lim_{t \to \infty} \left( \Lambda - \lambda_M^2 + K \sigma \theta \right) = 0
$$

Finally, from Equation (33), we have that:

$$
\lambda_M = \sqrt{\Lambda + K \sigma \theta}
$$

We can also estimate the magnetising flux from stator steady state values. For, note that:

$$
i \mu \frac{d}{dt} \mu i - i s \frac{d}{dt} \mu i = i q \frac{d}{dt} \mu i - i d \frac{d}{dt} \mu i - \sigma \left[ i q^2 + i d^2 \right]
$$

thus, replacing Equation (34) into Equation (28) and assuming that the stator dynamics is in steady state, we have:

$$
\lambda_M = \sqrt{\frac{L}{\theta}} \left( \frac{i q}{\mu} \left( i d i - i s i d \right) - L \sigma \left( i q^2 + i d^2 \right) \right)
$$

### 3.4 Accommodation of the detuning operation

From Equation (27) we have that in steady state:

$$
i_s = \frac{\lambda_M}{L_m}
$$

moreover, note that Equation (17) implies:

$$
\| i_\mu \| = \| i_\sigma \|
$$

thus, we can compute $i_\mu$ as:

$$
i_\mu = \sqrt{i_\sigma^2 + i_\mu^2 - i_\mu^2}
$$

Finally, from Equation (19), we have:

$$
\tau = \frac{i_\sigma^2 + i_\delta^2}{i_\delta i_\sigma}
$$

For accommodation of the detuning operation, we define a threshold $r_m > 0$ for the residual in such a way that the IFOC is detuned provided $|r| > r_m$. This threshold reduces the effect of noise and other non-modelled dynamics on the decision.

Once it is decided that the IFOC is detuned, the accommodation procedure waits for the system to achieve a steady state operation. Then it computes the new rotor time constant from Equation (36), and sends it to the IFOC scheme.
3.5 Simulations

We now validate the residual generator and the computation of the actual time rotor constant via numerical simulations. We consider an induction motor with the following parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Motor (3HP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_s, L_r (H)</td>
<td>0.3826, 0.3808</td>
</tr>
<tr>
<td>L_m (H)</td>
<td>0.3687</td>
</tr>
<tr>
<td>r_s, r_r (Ω)</td>
<td>1.34, 1.77</td>
</tr>
<tr>
<td>P</td>
<td>4</td>
</tr>
<tr>
<td>J (kg m²)</td>
<td>0.025</td>
</tr>
<tr>
<td>τ_L (Nm)</td>
<td>12</td>
</tr>
</tbody>
</table>

In the simulations we select Γ = 10.

In the following simulations, the accommodation procedure is not completed, we only compute the actual time rotor constant without correcting it on the IFOC scheme.

The residual behaviour is shown in Figure 3. In order to demonstrate that the residual is not asymptotically affected by changes of the load torque, at t = 3 sec the load torque is reduced by 50%. The residual converges to zero as shown in Figure 3.

**Figure 3** Residual behaviour r

To show that the residual detects changes of the rotor time constant at t = 3 sec, the time rotor constant is changed to twice the nominal value. Note that the residual detects the detuning condition by going to a non-zero equilibrium. At t = 10 sec the computed rotor time constant is τ_r = 0.56836, see Figure 4.
At $t = 6$ sec, the time rotor constant is changed to half of the nominal value, as we observe the detuning is detected. We compute the rotor time constant as $\tau_r = 0.14208$. Finally, at $t = 8$ sec, the time rotor constant is restored to its nominal value. We observe that the residual goes back to zero; in Figure 5 we present the mechanical rotor speed.

**Figure 4** Computed rotor time constant

**Figure 5** Mechanical rotor speed $\omega_m$
4 Broken rotor bar detection in IFOC driven squirrel cage induction motors

Industrial experience has shown that broken rotor bars can be a serious problem with certain induction motors with demanding work cycles. Although broken rotor bars do not initially cause an induction motor to fail, they can have serious secondary effects. The fault may result in broken parts of the bar hitting stator windings at high speed. This in turn can cause a serious damage to the induction motor; therefore, faulty rotor bars need to be detected as early as possible.

Broken rotor bars cause disturbances of the flux pattern in induction machines. These non-uniform magnetic field components affect machine torque and stator terminal quantities, and are thus detectable, in principle, by monitoring schemes. To date, different methods have been proposed for broken rotor bar detection. The most well-known approach is the non-model-based Motor Current Signature Analysis (MCSA) method (Thomson and Fenger, 2001). This method monitors the frequency spectrum of a single phase of the stator current for frequency components associated with broken rotor bars. The main disadvantage of the MCSA method is that it relies on the interpretation of the frequency components of the stator current spectrum, which are influenced by many factors, including variations in electric supply, and in static and dynamic load conditions. These conditions may lead to errors in the fault detection task (Benbouzid and Kliman, 2003). On the other hand, a practical advantage of MCSA is that only stator currents need to be measured. Efforts to eliminate the influence of load conditions have been presented for instance in Schoen and Thomas (1997), where it is shown that the direct component of the stator currents in a synchronous frame is not affected by load conditions; thus, it is proposed to monitor the spectrum of that stator current component. It turns out that in our proposed monitoring scheme, we monitor a state closely related to the direct component of the stator current. However, we discovered our signal selection using geometric techniques. Fuzzy logic (Ritchie et al., 1994) and neural network (Filippetti et al., 1995) techniques have also been proposed to handle load-related ambiguous frequency components. The MCSA method has been the main approach used for detecting broken rotor bars on induction motors operating in open-loop; however, spectral analysis techniques applicable under variable speed conditions have also been presented in the literature (e.g., Burnett et al., 1994; Watson and Elder, 1992).

In spite of the extensive work on broken rotor bar detection, model-based techniques have not received much attention. The main reasons are that fault-related induction motor parameters are not well known, and available models are quite complicated to be tractable with model-based fault detection techniques. However, by making a compromise between a better tracking of the fault-related signals (by using dynamic models) and a reduced domain of applicability of the results (due to assumptions about the induction motor parameters), model-based broken rotor bar detection techniques have been recently proposed. One such example is the Vienna Monitoring Method (VMM) presented in Kral et al. (2002). The VMM is based on the comparison of the computed electro-mechanical torque from two real-time machine models. A healthy induction motor leads to equal values computed by the two models, whereas a faulted induction motor excites the models in a different way, leading to a difference between computed torque values. This difference is used to determine the existence of broken rotor bars. The
VMM has one disadvantage, which is also present in our proposed monitoring scheme: variations on the time rotor constant cause the performance of the fault detection scheme to deteriorate.

4.1 Squirrel cage induction motor model with broken rotor bars

We now present an induction motor model with broken rotor bars. The proposed model is less detailed than the models presented, for instance, in Manolas and Tegopoulos (1999) and Williamson and Smith (1982). A novel feature is that the effect of broken rotor bars is taken into account by adding only one additional state to the classical induction motor model (presented for instance, in Krause et al., 1995). In this way, the tractability requirement is achieved.

The proposed model is based on the idea that the super-imposition of an extra set of rotor currents on those normally found in a healthy motor may account for the effect of broken rotor bars (Williamson and Smith, 1982). Our main assumptions are summarised as follows, see Figure 6.

- Due to high permeability of steel, magnetic fields exist only in the air gap and have radial direction $\vec{B}_r$, since the air gap is small relative to the inside diameter of the stator.
- The stator windings $a_s - a'_s$, $b_s - b'_s$ and $c_s - c'_s$ are identical in that each winding has the same resistance $r_s$ and the same number of turns. The rotor windings $a_r - a'_r$, $b_r - b'_r$ and $c_r - c'_r$ are identical in the same sense. All windings have sinusoidal distribution.
- The extra set of rotor currents (representing the broken bar) is included by adding an extra winding, denoted by $b_b - b'_b$, to the original rotor windings.
- Magnetic saturation, eddy-currents and friction losses are not included in our analysis.

Figure 6 Developed diagram of the cross-sectional view
In Figure 6, $\pi$, $\bar{\pi}$, $\tau$, and $\bar{\tau}$ denote the positive direction of the magnetic fluxes produced by each winding. $\odot$ indicates the positive direction of current. The angular displacement of the rotor relative to $\bar{\alpha}$ is denoted by $\phi_r$, the stator angular displacement relative to $\alpha$, is denoted $\phi_s$, while the rotor angular displacement relative to $\bar{\alpha}$ axis is denoted $\phi_s'$. The angular displacements $\theta_r$, $\phi_s$, and $\phi_s'$ are related as:

$$\phi_s = \phi_s + \phi_s'. $$

Following the modelling procedure of Krause et al. (1995), we have that the dynamic model of a squirrel cage induction motor with broken bars is described by:

$$
\begin{align*}
\dot{\lambda}_{abc} &= -R_i i_{abc} + v_{abc} \\
\dot{\lambda}_{abr} &= -R_i i_{abr} \\
\dot{\lambda}_b &= -r_b i_b 
\end{align*}
$$

where $\lambda_{abc}$, $\lambda_{abr}$ are the stator and rotor flux linkages, $i_{abc}$, $i_{abr}$ are the stator and rotor currents, $v_{abc}$ are the stator voltages, $\lambda_b$, $i_b$ are the broken bar-related flux linkage and current, $R_i = \text{diag}(r_i)$ is the rotor resistance matrix, with $r_i$ the rotor winding resistance, and $R_s = \text{diag}(r_s)$ is the stator resistance matrix. Flux linkages and currents are related as:

$$
\begin{bmatrix}
i_{abc} \\
i_{abr} \\
i_b
\end{bmatrix} =
\begin{bmatrix}
L_{s} & L_{sr} & L_{sb} \\
L_{sr} & L_{r} & L_{rb} \\
L_{sb} & L_{rb} & L_{b}
\end{bmatrix}
\begin{bmatrix}
\lambda_{abc} \\
\lambda_{abr} \\
\lambda_b
\end{bmatrix},
$$

where:

$$
\begin{align*}
L_{sb} &= L_{sb} \begin{bmatrix}
\cos(\theta_s) & \cos(\theta_s - c) & \cos(\theta_s + c)
\end{bmatrix}, \\
L_{sr} &= L_{sr} \begin{bmatrix}
\cos(\alpha) & \cos(\alpha - c) & \cos(\alpha + c)
\end{bmatrix}, \\
L_s &= \begin{bmatrix}
L_{ts} + L_{sr} & -\frac{1}{2}L_{sr} & -\frac{1}{2}L_{sr} \\
-\frac{1}{2}L_{sr} & L_{ts} + L_{sr} & -\frac{1}{2}L_{sr} \\
-\frac{1}{2}L_{sr} & -\frac{1}{2}L_{sr} & L_{ts} + L_{sr}
\end{bmatrix}, \\
L_{sr} &= \begin{bmatrix}
L_{ts} + L_{sr} & -\frac{1}{2}L_{sr} & -\frac{1}{2}L_{sr} \\
-\frac{1}{2}L_{sr} & L_{ts} + L_{sr} & -\frac{1}{2}L_{sr} \\
-\frac{1}{2}L_{sr} & -\frac{1}{2}L_{sr} & L_{ts} + L_{sr}
\end{bmatrix}, \\
L_{br} &= L_{br} \begin{bmatrix}
\cos(\theta_r) & \cos(\theta_r + c) & \cos(\theta_r - c)
\end{bmatrix}, \\
L_{bs} &= L_{bs} \begin{bmatrix}
\cos(\theta_s) & \cos(\theta_s + c) & \cos(\theta_s - c)
\end{bmatrix}, \\
L_{bs} &= L_{bs} \begin{bmatrix}
\cos(\theta_s + c) & \cos(\theta_s - c) & \cos(\theta_s)
\end{bmatrix}
\end{align*}
$$

with $\theta_s = \theta - \alpha$ and $c = \frac{\pi}{6}$.

In Equations (38) and (39), $L_{ls}$, $L_{ms}$ are the stator leakage and self inductance, $L_{ls}$, $L_{ms}$ are the rotor leakage and self inductance, $L_{sr} = L_{ms}$ the stator-rotor mutual inductance, $L_{br}$, $L_{bs}$ is the broken bar-related winding to stator and to rotor mutual inductance.
respectively, \( L_b \) is the broken bar-related winding self-inductance and \( \alpha \) is the angular position of the broken bar-related winding. Finally, the mechanical dynamics is described by:

\[
J \ddot{\omega}_m = \frac{P}{2} \frac{d}{d\theta} \left( i_{abc}^T L_{abc} i_{abc} + i_{abcr}^T L_{abcr} i_{abcr} \right) - r_L, \tag{40}
\]

Note that the broken bar-related winding inductances \( L_{br} \) and \( L_{bs} \), the resistance of the broken rotor bar-related winding \( r_b \) and the angular position \( \alpha \) are unknown parameters, since the number and the position of the broken rotor bars are unknown in advance.

### 4.2 Numerical validation of the proposed model

To validate the proposed induction motor with broken bars model, we need to compare our results with experimental measurements. However, in the absence of experimental studies, we will compare the results of the proposed model with the results of the squirrel cage induction motor model with broken rotor bars presented in Welsh (1988).

It is well known that broken bars result on sideband components around the fundamental of the stator currents at frequencies given by Benbouzid and Kliman (2003).

\[
f_s = (1 \pm 2s) f_s, \tag{41}
\]

where \( s \) is the per unit slip and \( f_s \) is the supply frequency. Moreover, experimental evidence has shown that when the amplitude of the broken rotor bar harmonics Equation (41) is over 50 dB smaller than the fundamental frequency component amplitude, the rotor may be considered healthy (Hirvonen, 1994). Thus, in our numerical studies we consider that the existence of harmonics at frequencies given by Equation (41) with amplitudes below 50 dB smaller than the fundamental as an indication that the model is able to reproduce the effect of broken rotor bars. We have considered two induction motors with parameters as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Motor 1 (3 HP)</th>
<th>Motor 2 (100 HP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{ls}, L_{lr} ) (H)</td>
<td>0.024, 0.013</td>
<td>0.0004, 0.0006</td>
</tr>
<tr>
<td>( L_{ms} ) (H)</td>
<td>0.2458</td>
<td>0.0144</td>
</tr>
<tr>
<td>( r_r, r_s ) (( \Omega ))</td>
<td>1.34, 1.77</td>
<td>0.037, 0.025</td>
</tr>
<tr>
<td>( P )</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( J ) (kg m^2)</td>
<td>0.025</td>
<td>0.863</td>
</tr>
<tr>
<td>( \eta_l ) (Nm)</td>
<td>12</td>
<td>270</td>
</tr>
<tr>
<td>Input voltage (V)</td>
<td>460 L-L</td>
<td>460 L-L</td>
</tr>
<tr>
<td>Rated current (A)</td>
<td>4</td>
<td>180</td>
</tr>
</tbody>
</table>

The induction motor model proposed in Welsh (1988) considers the rotor cage as a network of equally spaced loops, see Figure 7. Assuming that the rotor cage is symmetric, i.e., each rotor bar has the same resistance \( R_b \) and leakage inductance \( L_{db} \). The rotor bar parameters are obtained in terms of the standard single-phase equivalent-circuit model via a relationship that preserves the power transfered across the air-gap. Thus, the rotor bar parameters are given as follows:
with $L_{\text{mrb}}$, the stator rotor loop mutual inductance, $L_{\text{rbrb}}$, the rotor loop mutual inductance and $N_{\text{rb}}$, the number of rotor bars. Therefore, to implement this induction motor model, the number of rotor bars is needed. Note that in Equation (42), for given values of $L_{lr}$, $P$ and $L_{\text{ms}}$, there exists a minimum number of bars that give positive values for $L_{lrb}$, $L_{\text{mrb}}$ and $L_{\text{rbrb}}$. In our case, we have:

<table>
<thead>
<tr>
<th>Motor</th>
<th>Minimum number of rotor bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
</tbody>
</table>

Now, in order to obtain a more realistic number of rotor bars, we perform a limited survey of data from motors with similar characteristics. The collection of motors with characteristics similar to Motor 1 is shown in the next table:

<table>
<thead>
<tr>
<th>Rated power (HP)</th>
<th>Rated current (A)</th>
<th>Input voltage (V)</th>
<th>Number of rotor bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>2.1</td>
<td>380</td>
<td>30</td>
</tr>
<tr>
<td>2.95</td>
<td>5.3</td>
<td>380</td>
<td>28</td>
</tr>
<tr>
<td>2.01</td>
<td>3.5/6.1</td>
<td>380/220</td>
<td>28</td>
</tr>
<tr>
<td>1.48</td>
<td>2.8/4.9</td>
<td>380/220</td>
<td>22</td>
</tr>
</tbody>
</table>

Although the amount of data is not sufficient to make general conclusions, we can devise an estimate. From the table above, we can observe that there is a correlation between the rated current and the number of bars, thus, we select 28 bars for Motor 1. For Motor 2, we have:

<table>
<thead>
<tr>
<th>Rated power (HP)</th>
<th>Rated current (A)</th>
<th>Input voltage (V)</th>
<th>Number of rotor bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>210</td>
<td>440</td>
<td>58</td>
</tr>
</tbody>
</table>

so that we select 58 bars for Motor 2.
In order to break the bar in the model used in Welsh (1988), the loop currents of two adjacent loops are constrained to be equal, so that in the common bar, the total current is zero. With the considered parameters, we have for Motor 1: $s = 0.0113$ and $f_b = \{58.64, 61.35\}$, while for Motor 2, we have: $s = 0.0074$ and $f_b = \{59.11, 60.88\}$. As observed in Figure 8, the stator current has components at those frequencies with amplitude corresponding to a non-healthy induction motor. Another way to ‘break the bar’, presented in Menacer et al. (2004), is to increase the resistance of the bar that breaks. Ideally, the broken bar resistance should tend to infinity; however, this causes state continuity problems in the model. A workaround is to gradually increase the broken bar resistance. It is, however, difficult to compute the maximal broken bar resistance. In order to overcome this problem, we increase the resistance until a maximal value that does not cause numerical problems in our simulations. For instance, we increase the rotor bar resistance of bar 8 of Motor 1 from $5.0688 \Omega$ to $4034.29 \Omega$, which makes an increase of more than 700 times. This gives $s = 0.0170$ and $f_b = \{57.96, 62.03\}$, as shown in Figure 9. Finally, we perform some simulations with our proposed model with the following values for the broken bar-related parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Motor 1 (3HP)</th>
<th>Motor 2 (100HP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{lb} = L_{br}$ (H)</td>
<td>0.027</td>
<td>0.017</td>
</tr>
<tr>
<td>$L_b$ (H)</td>
<td>0.1104</td>
<td>1.97</td>
</tr>
<tr>
<td>$r_b$ (Ω)</td>
<td>0.90</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Figure 8  Spectral content of stator current $i_{st}$, the bar is broken by the constraint of equal adjacent loop currents

![Figure 8](image1)

Motor 1  
Motor 2

Figure 9  Spectral content of stator current $i_{st}$ in Motor 1, the bar is broken by increasing its resistance

![Figure 9](image2)

For our model, we have $s = 0.0164$, $f_b = \{58.02, 61.97\}$ for Motor 1 and $s = 0.0096$, $f_b = \{58.85, 61.14\}$ for Motor 2. Note that for the chosen parameters, the magnitude of the sidebands for Motor 2, barely reaches $-50$ dB. One of the differences we observe in our simulations concerning the method to include the broken bar is that the average mechanical speed changes. Thus, breaking the bar as in Welsh (1988) increases the average mechanical speed (less than 1 rd/sec); increasing the bar resistance decreases the average mechanical speed in the same proportion, while adding the extra winding also decreases the average mechanical speed. This causes the sidebands components to be located at different frequencies as shown in Figure 11. The magnitudes of the sidebands also have different values, but they are in the range of the non-healthy case.
4.3 Broken rotor bar detection

Now, we design a residual generator to detect broken rotor bars on an IFOC-driven squirrel cage induction motor. To this end, by considering $i_b$ as an externally generated signal, we express the induction motor dynamics Equation (1) in terms of a frame with phase locked with the direction of the rotor magnetising flux rotating at synchronous speed $\omega_f$. Thus, we have:
\[ \dot{\lambda}_q = -r_q i_q - \omega_r \dot{\lambda}_{db} + v_{qs} \]
\[ \dot{\lambda}_d = -r_d i_d + \omega_r \dot{\lambda}_{dq} + v_{ds} \]
\[ \dot{\lambda}_M = -\frac{1}{\tau_r} \lambda_M + \frac{L_m}{\tau_r} i_{qs} - \frac{L_m}{\tau_r} \frac{\cos(\theta_i - \alpha)}{\lambda_M} i_b \]  
(43)
\[ \omega = \omega_r + \frac{L_m}{\tau_r} i_{qs} - \frac{L_m}{\tau_r} \frac{\cos(\theta_i - \alpha)}{\lambda_M} i_b \]
\[ J \dot{\omega}_r = \frac{3P^2}{8} \left\{ \frac{L_m}{L_q} i_{qs} \dot{\lambda}_M + L_m \left[ \cos(\theta_i + \alpha) i_{ds} - \sin(\theta_i + \alpha) i_{qs} \right] i_b \right\} - T_L, \]
where \( \tau_r = \frac{L_m + L_m}{r_r} \) is the rotor time constant, \( \omega_r = \frac{2 \omega_p}{P} \) is the rotor angular speed and \( \theta_s = \theta_r - \theta_i \).

Assuming now that the induction motor is fed by current inverters with fast current controllers and considering an IFOC scheme, the induction motor dynamics that we consider for fault detection reads as:
\[ \dot{\lambda}_M = -\frac{1}{\tau_r} \lambda_M + \frac{L_m}{\tau_r} i_{ds} - \frac{L_m}{\tau_r} \sin(\theta_i - \alpha) i_b \]
\[ \dot{\omega}_r = \omega_r + \frac{L_m}{\tau_r} i_{qs} - \frac{L_m}{\tau_r} \frac{\cos(\theta_i - \alpha)}{\lambda_M} i_b \]
\[ J \dot{\omega}_r = \frac{3P^2}{8} \left\{ \frac{L_m}{L_q} i_{qs} \dot{\lambda}_M + L_m \left[ \cos(\theta_i + \alpha) i_{ds} - \sin(\theta_i + \alpha) i_{qs} \right] i_b \right\} - T_L, \]  
(44)
where \( i_d \) and \( i_q \) are the stator currents components controlled by the current controllers.

To write the rotor flux dynamics Equation (44) in terms of Equation (1), first we identify the target and nuisance faults. Since we want to design a broken rotor bar detector that is not influenced by load conditions, \( \tau_L \) and \( i_b \) in Equation (44) are identified as the nuisance and target fault modes, respectively, that is:
\[
\begin{bmatrix}
0 \\
0 \\
-1/J
\end{bmatrix}
\begin{bmatrix}
l_q \\
l_d
\end{bmatrix}
= \begin{bmatrix}
\frac{-L_m}{\tau_r} \sin(\theta_i - \alpha) \\
0 \\
\frac{3P^2 L_m}{8J} \left[ \cos(\theta_i + \alpha) i_{ds} - \sin(\theta_i + \alpha) i_{qs} \right]
\end{bmatrix}\
\begin{bmatrix}
l_q \\
l_d
\end{bmatrix}
+ \frac{L_m}{\tau_r} \frac{\cos(\theta_i - \alpha)}{\lambda_M} i_b\]
Moreover, we have:

\[
\begin{align*}
  f &= \begin{bmatrix} -\frac{1}{\tau_r} \lambda_M & \omega_r & 0 & \omega_r \end{bmatrix}^T, \\
  g &= \begin{bmatrix} \frac{L_m}{\tau_r} & 0 & 0 \\
 0 & 0 & \frac{3p^2}{8J} \lambda_M \\
 0 & 0 & \frac{L_m}{\tau_r \lambda_M} \end{bmatrix}.
\end{align*}
\]

From a practical point of view, it is desirable to design a residual generator using the rotor speed, as it is an easily measurable state. However, it can be shown that with the rotor speed as the output of Equation (44), the corresponding minimal unobservability distribution intersects the image of the nuisance fault signature, that is, the load condition effects cannot be removed from the residual. Note now that considering the rotor flux \( \lambda_M \) as the output of Equation (44), the minimal unobservability distribution \( S^* \) is computed as:

\[
S^* = \text{span}\begin{bmatrix} 0 & 0 & 0 \\
\frac{1}{\tau} & 0 & 0 \\
0 & \frac{1}{\tau} & 0 \\
\frac{1}{\tau} & 0 & 1 \end{bmatrix}.
\]

Thus, one has that Equation (3) is satisfied for \( \theta_r - \alpha \neq 0 \) and we can go further to find the diffeomorphism Equation (4). By inspection, we note that Equation (5), with \( w_1 = y \), reads as:

\[
\dot{\lambda}_M = -\frac{1}{\tau_r} \lambda_M + \frac{L_m}{\tau_r} i_b - \frac{L_m}{\tau_r} \sin(\theta_r - \alpha) i_b, \\
y = \lambda_M
\]

and as a result, we have that, Problem 1 is solvable with the residual generator dynamics described by:

\[
\begin{align*}
  \dot{\lambda}_M &= -\Gamma \lambda_M - \left(1 - \frac{1}{\tau_r - \Gamma} \right) \lambda_M + \frac{L_m}{\tau_r} i_b, \\
  r &= \dot{\lambda}_M - \lambda_M, \tag{47}
\end{align*}
\]

where \( \Gamma > 0 \). Furthermore, from the dynamics of \( r \) described by:

\[
r = -\Gamma r + \frac{L_m}{\tau_r} \sin(\theta_r - \alpha) i_b, \tag{48}
\]

it is possible to verify that Conditions 1 and 2 are satisfied, since for \( i_b = 0 \), the residual goes exponentially to zero, and is not affected by the nuisance fault (load torque). Moreover, for \( i_b \neq 0 \) the residual will move away from zero.
In Schoen and Thomas (1997), it is shown that an induction motor state that is not influenced by load conditions is the current $i_{ds}$, so that it is suggested to monitor the spectrum of $i_{ds}$. Note that in steady state $\lambda_m = L_m i_{ds}$. Thus, we arrived at the very same conclusion using non-linear geometric techniques.

Now, we verify the performance of the broken rotor bar detector via numerical simulations. In all simulations, we consider the induction motor model introduced in Welsh (1988). The residual behaviour for Motor 1 is shown in Figure 12. In order to verify that the residual is not affected asymptotically by changes on the load torque, at $t = 2 \, \text{sec}$, we increase the load torque by 50%. Note that the residual is not asymptotically affected. Now to show that the residual actually detects the effect of broken rotor bars, at $t = 4 \, \text{sec}$ ‘we break’ one rotor bar. Note that the residual, as predicted, detects this effect. The residual behaviour for Motor 2 is shown in Figure 13. At $t = 2 \, \text{sec}$, we reduce the load torque by 50%. Note that the residual is not asymptotically affected. At $t = 4 \, \text{sec}$, one rotor bar is broken. As predicted by our computations, the residual reacts to the target fault. However, note that if the rotor time constant is not exactly known, deviations from the value used in the residual generator will produce a reaction of the fault detector. Since changes on the rotor time constant are mainly due to the rise of the temperature of the motor, the reaction of the fault detector to this mismatch should be slow. This problem also occurs in the VMM, as it is assumed that the rotor time constant is known exactly. Noisy measurements can also disturb the detector’s performance. However, our initial analysis indicate that it is possible to distinguish between noisy measurements and broken rotor bar-driven residuals.

**Figure 12** Residual behaviour for Motor 1
Note that the limitations of the developed induction motor model affect the fault detector scheme. For instance, since we consider ideally distributed stator and rotor windings, it is not possible to determine the influence of other current harmonics on the residual.

5 Conclusions

The paper reviewed model-based fault detection techniques in the analytic redundancy framework. Our review covered approaches from the detection filter of Beard and Jones to the non-linear unknown input observers. We outlined the non-linear unknown input observer technique in order to solve fault detection problems in induction motors.

Next, we presented a monitoring scheme to detect detuning operation on indirect field-oriented controlled current-fed induction motors. The key for fault detection was the development of a model, based on the detuning interpretation introduced in Mijalkovic (2002), that expresses the detuning effect in terms of the difference between the real and the estimated rotating frames, and the selection, based on techniques from differential geometry theory, of the induction motor state to monitor. It is also shown how the fault can be accommodated. Numerical simulations validate the monitoring scheme and show that the monitoring scheme is not affected by changes on the load torque.

We also developed a simplified model for a squirrel cage induction motor that includes broken rotor bars effects. Relying on differential geometry techniques, we have proposed a model-based solution to the broken rotor bar detection problem on IFOC-driven squirrel cage induction motors. We show that load torque conditions will not lead to errors in the detection, as the fault detector is not affected by such condition. Numerical simulations of very different induction motors validate the model and the performance of the monitoring scheme.

Our analysis suggests that model-based fault detection techniques will have an important role to play in emerging fault-tolerant electric drive systems. In such applications, they will complement the model-free techniques in cases where accurate but tractable models are available.
References


Dynamical models for fault detection in squirrel cage induction motors


**Note**

1 All the way through vectors are denoted as $f_{abc} = \begin{bmatrix} f_a & f_b & f_c \end{bmatrix}^T$. 