Full-Duplex Cooperative Diversity with Alamouti Space-Time Code

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Abstract—In this letter, we present a new distributed full-duplex (FD) Alamouti scheme for a three-node cooperative network. The proposed scheme exploits the FD operation at the relay node and forms the Alamouti codewords in three block periods. We characterize the outage probability and the diversity-multiplexing tradeoff of the investigated FD scheme. Our analysis demonstrates that as the strength of the LI decreases and the spectral efficiency increases, the FD Alamouti scheme significantly outperforms conventional half-duplex designs.

Index Terms—Relay channel, full-duplex, Alamouti code, outage probability, diversity-multiplexing tradeoff.

I. INTRODUCTION

COOPERATIVE communication is an efficient technique to provide spatial diversity in single-antenna systems. The key idea behind the concept is to allow single-antenna terminals to cooperate in order to create “virtual” transmit/receive multiple antenna configurations [1]. Cooperative communication has been extensively studied for half-duplex (HD) relaying [2]. For example, an efficient HD cooperative protocol which implements the Alamouti code [3] in a distributed fashion for a three-node relay channel has been proposed in [4]. However, the rate of this scheme is limited to 0.5 symbols per channel use since the deployment of the Alamouti code requires four symbol periods. On the other hand, full-duplex (FD) relaying has the potential to improve the spectral efficiency [5], [6], [7], [8]. In FD, the relay can simultaneously transmit and receive but at the cost of loopback interference (LI) from the relay output to the relay input [6], [7]. While most work on FD relaying focus on LI mitigation [8], [9], which is the main bottleneck, recent studies also investigate space-time coding techniques that exploit the LI and the associated artificial multi-tap channel, e.g., [10], [11].

In this work, we study the distributed implementation of the Alamouti code for a three-node cooperative network with FD operation at the relay node. We focus on the Alamouti code due to its simplicity and its high impact on the modern wireless communications standards such as LTE-Advanced. In addition, this work is among the first that studies the beneficial combination of direct and relaying link in a FD context; most of the existing literature treats the direct link as interference to the relaying link. FD relaying allows the distributed implementation of the Alamouti scheme in three block periods and we show that provides significant spectral efficiency gains. In particular, we characterize the outage probability and the diversity-multiplexing tradeoff (DMT) [12] of the new FD-based scheme; the impact of the LI on the DMT is also elucidated. Analytical and simulation results demonstrate that for low LI degradation as well as for high spectral efficiencies, the FD-based Alamouti scheme significantly outperforms conventional HD designs with single/dual antennas at the relay.

Notation: Boldface letters indicate vectors (lower case) or matrices (upper case). The superscripts $(\cdot)^T$, $(\cdot)^*$, denote the transpose and the conjugate, respectively. A circularly symmetric complex Gaussian random variable $z$ with mean $\mu$ and variance $\sigma^2$ is represented as $z \sim \mathcal{CN}(\mu, \sigma^2)$. The identity matrix of size $M$, and the zero matrix of size $m \times n$, are denoted by $I_M$ and $0_{m \times n}$, respectively; $\log(\cdot)$ denotes the logarithm of base 2, $E\{\cdot\}$ is the expectation operator and $\|\cdot\|$ denotes the Euclidean norm.

II. SYSTEM MODEL

We consider a decode-and-forward (DF) relay channel consisting of one source, $S$, one relay, $R$, and one destination, $D$. Both $S$ and $D$ have a single antenna each, while $R$ has two antennas (with FD, one antenna for reception and one antenna for transmission). In order to implement the distributed Alamouti code and facilitate decoding at $R$, the source transmission is ordered into frames and blocks. Each block contains $L$ continuous symbols1 whereas three and four blocks are arranged into a frame for FD and HD, respectively. The system employs the distributed Alamouti code when $R$ can perfectly decode the source’s signal; else a direct transmission is enforced [4]. $R$ decodes at the end of each block and FD operation allows to receive and transmit the $(i+1)$-th block and the $i$-th block, respectively, in the same time and frequency band (delay of one block) [11]. We model the channels between the nodes as flat Rayleigh block-fading; static for $3L$ (FD operation) and $4L$ (HD operation) symbols, respectively. Let $h_{sd} \sim \mathcal{CN}(0, \eta_{sd})$, $h_{sr} \sim \mathcal{CN}(0, \eta_{sr})$, and $h_{rd} \sim \mathcal{CN}(0, \eta_{rd})$ represent the circularly symmetric complex Gaussian channel coefficients from $S$ to $D$, $S$ to $R$ and $R$ to $D$, respectively. Moreover, $h_{rr} \sim \mathcal{CN}(0, \eta_{rr}, \text{SNR}^{-\mu})$ represents the LI channel from the relay output to the relay input, where SNR is the signal-to-noise ratio (SNR) for each symbol and the variable $\mu \geq 0$ captures how the residual interference scales with SNR [11]. Power allocation is not considered and in the case where both $S$ and $R$ are active, the total transmit power, $P$, is equally divided.

III. DISTRIBUTED ALAMOUTI PROTOCOLS

A. FD Alamouti Protocol

Let $\mathbf{x}_1 = [x_1(1), \ldots, x_1(L)]^T$ and $\mathbf{x}_2 = [x_2(1), \ldots, x_2(L)]^T$ denote the vectors that contain the

1The adopted structure empowers the DF relay with error correction capability such as cyclic redundancy check on a per block basis.

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information symbols with $\mathbb{E}\{\|\mathbf{x}_k\|^2\} = LP$, $k = 1, 2$. In the first phase, $\mathcal{S}$ transmits the symbol vector $\mathbf{x}_1$ and the $l$-th received symbol at $\mathcal{R}$ and $\mathcal{T}$ are given by

\begin{align}
y_{l1}(l) &= \sqrt{\text{SNR}} h_{sr} x_1(l) + w_{l1}(l), \\
y_{l2}(l) &= \sqrt{\text{SNR}} h_{sd} x_2(l) + w_{l2}(l),
\end{align}

where $l = 1, \ldots, L$. Finally, if $\mathcal{R}$ cannot decode the signal from $\mathcal{S}$ correctly ($\mathcal{S} - \mathcal{R}$ link is in outage), the Alamouti code cannot be implemented with FD/HD operation and the system operates in the conventional single-input single-output (SISO) mode (direct link $\mathcal{S} - \mathcal{T}$).

IV. PERFORMANCE ANALYSIS

A. Outage Probability

1) FD Alamouti Protocol: We study the outage probability of the proposed distributed FD Alamouti scheme for a spectral efficiency $R$ in BPCU. The switching between the two protocols’ cooperative/SISO modes depends on the outage status of the $\mathcal{S} - \mathcal{R}$ link. Recall that $\mathcal{R}$ employs the cooperative mode only when it can decode both symbols transmitted by $\mathcal{S}$ during the first two phases of the protocol. The outage probability of the $\mathcal{S} - \mathcal{R}$ link is dominated by the source transmission in the second phase since it suffers from LI. Therefore, we can express the outage probability of the FD Alamouti scheme as

\begin{align}
P_{\text{out}} = P_{\text{FDAL}}(R)(1 - \pi(R)) + P_{\text{SISO}}(R)\pi(R),
\end{align}

where $\pi(R)$ denotes the outage probability for the $\mathcal{S} - \mathcal{R}$ link, $P_{\text{SISO}}(R)$ and $P_{\text{FDAL}}(R)$ are the outage probabilities for the SISO mode and the cooperative mode of the protocol, respectively.

We can write $P_{\text{SISO}}(R) = 1 - e^{-\frac{R}{h_{sd}}}$. Moreover, $\pi(R)$ can be obtained as

\begin{align}
\pi(R) &= 1 - \frac{\frac{3}{2} + 2R}{\frac{3}{2} + 2R - \frac{3}{2}h_{sr}}.
\end{align}

Deriving an exact expression for $P_{\text{out}}$ appears intractable since $P_{\text{FDAL}}(R)$ does not admit a closed-form solution. However, according to $C_{\text{FB}} < \frac{1}{3} \log \left( \left( \frac{1 + \frac{3}{2}h_{sr}}{h_{sr}} \right) \left( \frac{2h_{sr}}{2n_{sr}} \right) \right)$ and $C_{\text{FB}} > \frac{1}{3} \log \left( \left( \frac{1 + \frac{3}{2}h_{sr}}{h_{sr}} \right) \left( \frac{2h_{sr}}{2n_{sr}} \right) \right)$, we can derive a
lower and an upper bound for $P_{\text{FDAL}}(R)$. To this end, note that we can write

$$\mathbb{P}\{a|h_{sd}|^2 + |h_{rd}|^2 < z\} \overset{(1)}{=} \frac{\gamma_d}{\gamma_d} \left(1 + \frac{z}{\gamma_d}\right),$$

$$\overset{(2)}{=} \frac{e^{-\frac{a}{\eta_{sd}}} - 1}{\frac{1}{\eta_{sd}} - 1} - \frac{e^{-\frac{a}{\eta_{rd}}} - 1}{\frac{1}{\eta_{rd}} - 1},$$

(20)

for $\overset{(1)}{=}$; $\eta_{sd} = \frac{1}{2} \eta_{rd}$ and $\overset{(2)}{=}$; $\eta_{sd} \neq \frac{1}{2} \eta_{rd}$. The lower and upper bound for $P_{\text{FDAL}}(R)$ now follows with $z = \frac{\alpha R}{\frac{1}{\alpha} - 1}, \alpha = 3$ and $\alpha = 1$, respectively. Now substituting $P_{\text{SISO}}(R), \pi(R)$ and $P_{\text{FDAL}}(R)$ into (15), we can express the corresponding lower and upper bound of $P_{\text{out}}$ in closed-form.

2) HD Alamouti Protocol I: We can express the outage probability as [4, Eq. (6)]

$$P_{\text{out}} = P_{\text{HD1}}(R)(1 - \omega_1(R)) + P_{\text{SISO}}(R)\omega_1(R),$$

(21)

with

$$P_{\text{HD1}}(R) = \mathbb{P}\{C_{\text{HD1}} < R\},$$

$$\omega_1(R) = \mathbb{P}\{\log (1 + \text{snr})|h_{sr}|^2) < 2R\}.$$  

(22)

Now $P_{\text{HD1}}(R)$ can be evaluated with $z = \frac{\alpha R}{\frac{1}{\alpha} - 1}$ and $\alpha = 3$ in (20) and $\omega_1(R) = 1 - e^{-\frac{2R}{3\eta}}$. Substituting these results into (21) we can evaluate the exact outage probability in closed-form.

3) HD Alamouti Protocol II: We can express the outage probability as

$$P_{\text{out}} = P_{\text{HD2}}(R)(1 - \omega_2(R)) + P_{\text{SISO}}(R)\omega_2(R),$$

(24)

with

$$P_{\text{HD2}}(R) = \mathbb{P}\{C_{\text{HD2}} < R\},$$

$$\omega_2(R) = \mathbb{P}\{\log (1 + \text{snr}(|h_{sr}(1)|^2 + |h_{sr}(2)|^2) < 2R\}.$$  

(25)

$$P_{\text{HD2}}(R)$$ can be evaluated as

$$P_{\text{HD2}}(R) \overset{(1)}{=} 1 - e^{-\frac{z}{\eta_{sd}}} - e^{-\frac{z}{\eta_{rd}}} \left(1 + \frac{z^2}{\eta_{sd}}\right),$$

$$\overset{(2)}{=} 1 - \frac{e^{-\frac{z}{\eta_{sd}}} - 1}{\frac{1}{\eta_{sd}} - 1} - \frac{e^{-\frac{z}{\eta_{rd}}} - 1}{\frac{1}{\eta_{rd}} - 1},$$

(27)

with $z = \frac{2\alpha R}{3\eta_{sr}}$. In (27) $\overset{(1)}{=}$; for $\eta_{sd} = \frac{1}{2} \eta_{rd}$ and $\overset{(2)}{=}$; for $\eta_{sd} \neq \frac{1}{2} \eta_{rd}$. We can also write $\omega_2(R) = 1 - e^{-\frac{2\alpha R}{3\eta_{sr}}\left(1 + \frac{z^2}{3\eta_{sr}}\right)}$.

Now substituting (27), $\omega_2(R)$ with $P_{\text{SISO}}(R)$ into (24) yields the exact outage probability.

B. Diversity-Multiplexing Tradeoff

1) FD Alamouti Protocol: DMT is an effective information-theoretic criterion to evaluate the spectral efficiency of cooperative transmission protocols and is the performance metric adopted in this work [12]. A diversity gain $d(r)$ is achieved at multiplexing gain $r$ if

$$\lim_{\text{snr} \to \infty} \frac{\log P_{\text{out}}(r \log \text{snr})}{\log \text{snr}} = -d(r).$$

(28)

We define $u_1 = -\lim_{\text{snr} \to \infty} \frac{\log |h_{sd}|^2}{\log \text{snr}} \implies u_1 = \text{snr}^{-u_1}$, where $\text{snr}^{-u}$ denotes an asymptotic behavior when $\text{snr} \to \infty$; similarly we have $|h_{sr}|^2 \approx \text{snr}^{-u_2}, |h_{rd}|^2 \approx \text{snr}^{-u_3}$ and $|h_{rr}|^2 \approx \text{snr}^{-u_4}$. By using the outage expression given in (15) and by setting $R = r \log \text{snr}$, we can derive the DMT of the proposed FD Alamouti scheme. More specifically, by following the basic techniques in [12], we have

$$P_{\text{SISO}}(r \log \text{snr}) \approx \mathbb{P}\{\log (\text{snr}^{-u_1}) < r \log \text{snr}\} \approx \mathbb{P}\{1 - u_1 < r\} \approx \text{snr}^{-d_{\text{SISO}}},$$

(29)

with $d_{\text{SISO}} = \inf \{u_1\} = 1 - r$. Now consider the lower bound of $P_{\text{FDAL}}(R)$ and we can write

$$P_{\text{FDAL}}(r \log \text{snr})$$

$$> \mathbb{P}\left\{\frac{1}{3} \log \left(\frac{1 + \text{snr}}{2} (3|h_{sd}|^2 + |h_{rd}|^2)^2\right)^2 < r \log \text{snr}\right\}$$

$$\approx \mathbb{P}\left\{\frac{1}{3} \log (\text{snr}^{-u_1} + \text{snr}^{-u_2})^2 < r \log \text{snr}\right\}$$

$$\approx \mathbb{P}\{\max \{1 - u_1, 1 - u_3\} < \frac{3}{4} r\} \approx \text{snr}^{-d_{\text{FDAL}}},$$

(30)

with $d_{\text{FDAL}} = \inf \{u_1 + u_3\} = 2 (1 - \frac{3}{4})^+ = (\mu + 3)^+ = \max(0, \mu)$. Moreover, using a similar approach, we can show that

$$\text{snr}^{-d_{\text{FDAL}}}$$

$$= \mathbb{P}\left\{\frac{1}{3} \log \left(\frac{1 + \text{snr}}{2} (|h_{sd}|^2 + |h_{rd}|^2)^2\right)^2 < r \log \text{snr}\right\}$$

$$\approx \text{snr}^{-d_{\text{SISO}}}.$$  

(31)

Now from (30) and (31) we can conclude that $P_{\text{FDAL}}(r \log \text{snr}) \approx \text{snr}^{-d_{\text{FDAL}}}$. Finally, we have

$$\pi(r \log \text{snr}) \approx \mathbb{P}\left\{\frac{2}{3} \log \frac{\text{snr}^{-u_2}}{\text{snr}^{-u_2}} < r \log \text{snr}\right\}$$

$$\approx \mathbb{P}\left\{\log \text{snr}^{-u_2} - u_2 < 3 \frac{1}{2} \log \text{snr}\right\}$$

$$\approx \mathbb{P}\{\mu + u_2 - u_2 < \frac{3}{2} r\} \approx \text{snr}^{-d_{\pi}}.$$  

(32)

with $d_\pi = \inf \{u_2\} = (\mu + \frac{3}{2})^+$. Therefore, the DMT performance of the FD Alamouti scheme can be written as

$$d = \min\{d_{\text{FDAL}}, d_{\pi} + d_{\text{SISO}}\}$$

$$= \begin{cases} (\mu + \frac{3}{2})^+ + (1 - r), & \mu < 1 - \frac{3}{2} r, \\ 2 (1 - \frac{3}{2} r)^+, & \text{else} \end{cases}$$

(33)

2) HD Alamouti Protocol I: We note that the DMT of the HD Alamouti scheme is given by $d = 2(1 - 2r)^+$ [4].

3) HD Alamouti Protocol II: Let $|h_{sr}(k)|^2 \approx \text{snr}^{-u_3}$ and $|h_{rd}(k)|^2 \approx \text{snr}^{-u_3}$, for $k = 1, 2$. We can write

$$P_{\text{HD2}}(r \log \text{snr})$$

$$\approx \mathbb{P}\{\log (|\text{snr}^{-u_1} + \frac{1}{2} \text{snr}^{-u_3}) < 2r \log \text{snr}\}$$

$$\approx \mathbb{P}\{\max (1 - u_1, 1 - u_3, 1 - u_3) < 2r\} \approx \text{snr}^{-d_{\text{HD2}}}.$$  

(34)
with $d_{\text{HD2AL}} = \inf \{ u_1 + u_{31} + u_{32} \} = 3(1 - 2r)$. Similarly, $\omega_2(r \log \text{snr}) \leq \log \{(\text{snr}^{1-u_{21}} + \text{snr}^{1-u_{22}}) < 2r \log \text{snr} \}$ 
\begin{equation}
\omega_2 \leq \inf \{ u_{21} + u_{22} \} = 2(1 - 2r),
\end{equation}
where $d_{\omega_2} = \inf \{ u_{21} + u_{22} \} = 2(1 - 2r)$. Therefore, we can express the DMT performance of the HD Alamouti Protocol II as
\begin{equation}
d = \min \{d_{\text{HD2AL}}, d_{\omega_2} + d_{\text{SISO}} \} = 3(1 - 2r).
\end{equation}
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\section{V. Numerical Results}

In this section, we present some numerical results to verify the performance of the proposed FD Alamouti protocol. In Fig. 1 we compare the outage probability of the proposed scheme with the HD Alamouti Protocols I and II with $R = 3$ BPCU and $\mu = \{1, 0.8, 0.5\}$ (different $\mu$ represent different levels of LI). Firstly, we observe that as $\mu$ increases, FD Alamouti becomes an interesting approach. FD Alamouti outperforms HD Alamouti Protocol I with a coding gain about 1 dB for $\mu = 0.8$. Secondly, as $\mu$ decreases, FD Alamouti suffers from a diversity loss. However, it is worth noting that also for the case with $\mu = 0.5$, the FD Alamouti outperforms the HD Alamouti Protocol I at moderate SNRs (i.e., 20 dB). On the other hand, we can see that HD Alamouti Protocol II enjoys a diversity gain equal to three but at the cost of a higher complexity (2 RF chain pairs). In the same figure, we plot the theoretical results, which validate our analysis. In Fig. 2 we plot the DMT performance of the FD Alamouti and the HD Alamouti (Protocols I, II) for different levels of LI. As $\mu$ and $r$ increase, the FD Alamouti protocol outperforms its HD Alamouti protocols. In the extreme case of $\mu = 1$, DMT of the FD Alamouti Protocol I in all the multiplexing gain regions, while outperforms HD Alamouti Protocol II for $r > 0.35$ BPCU.

\section{VI. Conclusion}

We have investigated a distributed implementation of the Alamouti code for a FD cooperative network with direct link. The new scheme achieves a transmission of the Alamouti codewords in three block periods in contrast to the HD schemes that require four block periods. We analytically characterized the outage probability and the DMT performance of the proposed FD protocol and compared with RF/antenna conserved HD schemes. The proposed FD scheme becomes an interesting solution over the HD counterparts as the spectral efficiency increases and the strength of the LI decreases.

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\section{REFERENCES}