## Kat' óıkov Epyaqía 2

1. Prove the following simplification theorems using the first fifteen axioms/laws of Boolean Algebra:
a. $(X+Y)\left(X+Y^{\prime}\right)=X$
b. $X(X+Y)=X$
c. $\left(X+Y Y^{\prime}\right) Y=X Y$
d. $(X+Y)\left(X^{\prime}+Z\right)=X Z+X^{\prime} Y$
2. Find the complement of the following functions:
a. $f(A, B, C, D)=\left(A+(B C D)^{\prime}\right)\left((A D)^{\prime}+B\left(C^{\prime}+A\right)\right)$
b. $f(A, B, C, D)=A B^{\prime} C+\left(A^{\prime}+B+D\right)\left(A B D^{\prime}+B^{\prime}\right)$
3. Simplify the following Boolean functions using the laws and theorems of Boolean Algebra. Write the particular law or theorem you are using at each step. For each simplified function you derive, how many literals does it have?
a. $f(X, Y)=X Y+X Y^{\prime}$
b. $f(X, Y)=(X+Y)\left(X+Y^{\prime}\right)$
c. $f(X, Y, Z)=Y Z^{\prime}+X^{\prime} Y Z+X Y Z$
d. $f(X, Y, Z)=(X+Y)\left(X^{\prime}+Y+Z\right)\left(X^{\prime}+Y+Z^{\prime}\right)$
e. $f(W, X, Y, Z)=X+X Y Z+X ' Y Z+X ' Y+W X+W \prime X$
4. Consider the function $f(A, B, C, D)=\left(A D+A^{\prime} C\right)\left(B^{\prime}\left(C+B D^{\prime}\right)\right)$.
a. Draw its digital circuit using AND, OR, and NOT gates.
b. Using Boolean Algebra, put the function into its minimized form and draw the resulting digital circuit.
5. Consider the function $f(A, B, C, D)=\sum m(0,1,2,7,8,9,10,15)$.
a. Write this as a Boolean expression in canonical minterm (SoP) form.
b. Rewrite the expression in canonical maxterm (PoS) form.
c. Write the complement of $f$ in "little $m$ " notation and as a canonical minterm expression.
d. Write the complement of $f$ in "big M" notation and as a canonical maxterm expression.
6. Given the following function in product of sums form, not necessarily minimized: $F(W, X, Y, Z)=\left(W+X^{\prime}+Y^{\prime}\right)\left(W^{\prime}+Z^{\prime}\right)(W+Y)$
a. Express the function in the canonical sum of products form. Use "little m" notation.
b. Re-express the function in minimized sum of products form.
c. Express $F^{\prime}$ in minimized sum of products form.
d. Re-express $\mathrm{F}^{\prime}$ in minimized product of sums form.
7. Find the simplest form of the functions described by the following Kmaps:


8. Use Karnaugh maps (K-maps) to simplify the following functions in sum of products form. How many literals appear in your minimized solutions?
a. $f(W, X, Y, Z)=\Pi M(0,1,6,7)$
b. $f(W, X, Y, Z)=\Pi M(1,3,7,9,11,15)$
c. $f(V, W, X, Y, Z)=\Pi M(0,4,18,19,22,23,25,29)$
d. $f(A, B, C, D)=\sum m(0,2,4,6)$
9. Optimize the following Boolean functions together with the don't care conditions d, using K-maps. List all prime implicants and all essential prime implicants.
a. $f(A, B, C)=\sum m(3,5,6)$

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d(A, B, C)=\sum m(0,7)
$$

b. $f(W, X, Y, Z)=\sum m(0,2,4,5,8,14,15)$

$$
d(W, X, Y, Z)=\sum m(7,10,13)
$$

c. $f(A, B, C, D)=\sum m(4,6,7,8,12,15)$
$d(A, B, C, D)=\sum m(2,3,5,10,11,14)$
10. Given the function $f(A, B, C, D)=\sum[m(1,5,7,8,9,13,15)+d(4,14)]$, find the minimum sum of products form using the Quinne-McCluskey method.

