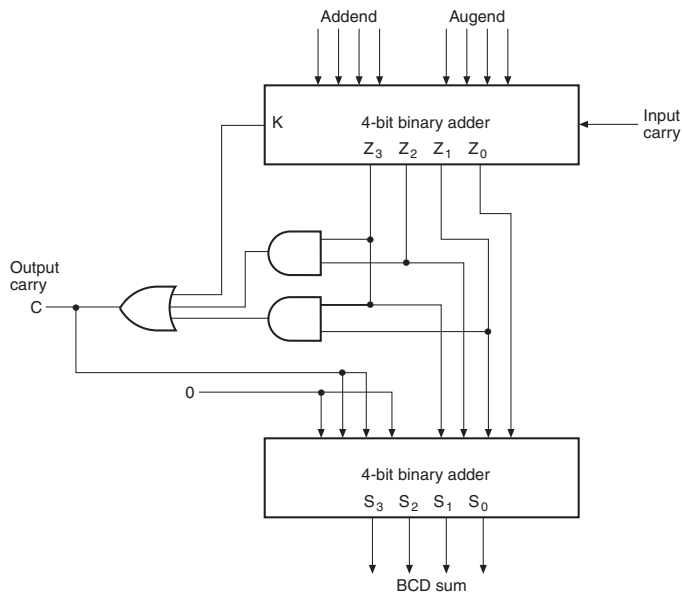# DECIMAL ARITHMETIC

S elected topics not covered in the fourth edition of *Logic and Computer Design Fundamentals* are provided here for optional coverage and for self-study, if desired. This material fits well with the desired coverage in some programs but not may not fit within others due to time constraints or local preferences. This supplement is referenced in Chapter 4 as a part of the coverage of Arithmetic Functions and Circuits.

Computers, calculator, or other specialized digital systems that perform arithmetic operations directly in the decimal number system represent decimal numbers in binary coded form. An adder for such a system must employ arithmetic circuits that accept coded decimal numbers and present results in the same code. For binary addition, it is sufficient to consider a pair of significant bits together with a previous carry. A decimal adder requires a minimum of nine inputs and five outputs, since four bits are required to code each decimal digit and the circuit must have an input and output carry. There is a wide variety of possible decimal adder circuits, depending upon the code used to represent the decimal digits. Here we consider a decimal adder for the BCD code.

The rules for BCD addition were presented in Section 1-5. First, the BCD digits are added as if they were two 4-bit binary numbers. When the binary sum is less than or equal to 1001 (decimal 9), the corresponding BCD digit sum is correct. However, when the binary sum is greater than 1001, we obtain an invalid BCD result. The addition of binary 0110 (decimal 6) to the binary sum converts it to the correct BCD representation and also produces an output carry as required. Consider the addition of two decimal digits in BCD with an input carry. Since each digit does not exceed 9, the sum cannot be greater than $9 + 9 + 1 = 19$.

The logic circuit that checks for the necessary BCD correction can be derived by detecting the occurrence of the binary numbers from 1010 through 10011 (decimal 10 through 19). It is obvious that a correction is needed when the binary sum

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□ **FIGURE 1**
Block Diagram of BCD Adder

has an output carry. This condition occurs when the sum is greater than or equal to 16. The other six combinations, from 1010 through 1111, that need a correction have a 1 in the most significant position and a 1 in the second or third significant position. A BCD adder that adds two BCD digits and produces a sum digit in BCD is shown in Figure 1. It has two 4-bit binary adders and correction logic. The two decimal digits, together with an input carry, are added in the first 4-bit binary adder, to produce the binary sum. The condition for correction can be expressed by the Boolean function

$$C = K + Z_1 Z_3 + Z_2 Z_3$$

Here C is the output carry from the BCD adder, and K is the output carry from the first binary adder. The two terms with the Z variables detect the binary outputs from 1010 through 1111. When the BCD carry is equal to 0, nothing is added to the binary sum. This condition occurs if the sum of the two digits plus the input carry is less than or equal to binary 1001. When the output carry is equal to 1, binary 0110 is added to the binary sum through the second 4-bit adder. This condition occurs when the sum is greater than or equal to 1010. Any output carry from the second binary adder can be neglected. A decimal parallel adder that adds two n-digit decimal numbers needs n BCD adders. The output carry from each adder must be connected to the input carry of the adder in the next higher position.

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Use of Complements in Decimal

The formation of the 9's and 10's complement was briefly discussed earlier. The following two examples illustrate the formation of complements in decimal:

The 9's complement of 546700 is $999999 - 546700 = 453299$.

The 10's complement of 546700 is $1000000 - 546700 = 453300$.

Thus, as with binary, the decimal 10's complement is obtained by adding 1 to the 9's complement value.

Since 10^n is a number represented by a 1 followed by n 0's, the 10's complement of N, $10^n - N$, also can be formed by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and then subtracting all higher significant digits from 9. Accordingly, the 10's complement of 234500 is 765500 and is obtained by leaving the two zeros unchanged, subtracting 5 from 10, and subtracting the other three digits from 9.

The next two examples apply the use of complements to unsigned decimal subtraction.

• EXAMPLE 1 Unsigned Decimal Subtraction Using 10's Complement

Using 10's complement, perform the subtraction $72532 - 3250$.

$$M = 72532$$
$$\text{10's complement of } N = +\ \underline{96750}$$
$$\text{Sum} = 169282$$
$$\text{Discard end carry } 10^5 = -\ \underline{100000}$$
$$Answer = 69282$$

Note that M has five digits and N has only four digits. To perform the subtraction $M - N$, both numbers must have the same number of digits, so we write N as 03250. Taking the 10's complement of N produces a 9 in the most significant position. The occurrence of the end carry in the addition signifies that $M > N$ and the result is positive. •

• EXAMPLE 2 Unsigned Decimal Subtraction Using 10's Complement

Using 10's complement, perform the subtraction $3250 - 72532$.

$$M = 03250$$
$$\text{10's complement of } N = +\ \underline{27468}$$
$$\text{Sum} = 30718$$

There is no end carry.

$$Answer: -\ (\text{10's complement of } 30718) = -69282$$

Note that since $3250 < 72532$, the result is negative. •

The procedure with end-around carry, as used in binary, is also applicable for subtracting unsigned decimal numbers with 9's complement.

PROBLEMS

The plus (+) indicates a more advanced problem.

1. Obtain the 9's complement of the following 8-digit decimal numbers: 21539740, 01011101, 90529960, and 00000000.

2. Obtain the 10's complement of the following 6-digit decimal numbers: 692310, 004567, 100101, and 000000.

3. Perform the indicated subtraction with the following unsigned decimal numbers by taking the 10's complement of the subtrahend:
(a) $6789 - 2345$ **(b)** $2007 - 2288$ **(c)** $40 - 110$ **(d)** $1940 - 699$

4. *Design two simplified combinational circuits that generates the 9's complement of (a) a BCD digit and (b) an excess-3 digit. (c) Compare the gate and literal counts of the two circuits. Assume in both cases that input combinations not corresponding to decimal digits give don't care outputs.

5. Construct a BCD adder-subtractor using the BCD adder from Figure 1 and a 9's complementer, as well as other logic or functional blocks, as necessary. Use block diagrams for the components, showing only inputs and outputs where possible.

6. It is necessary to design a decimal adder for digits represented in the excess-3 code. Show that the correction after adding two digits with a 4-bit binary adder is as follows:

(a) The output carry is equal to the carry from the binary adder.
(b) If the output carry = 1, then add 0011.
(c) If the output carry = 0, then add 1101.
(d) Construct the decimal adder with two 4-bit adders and an inverter.