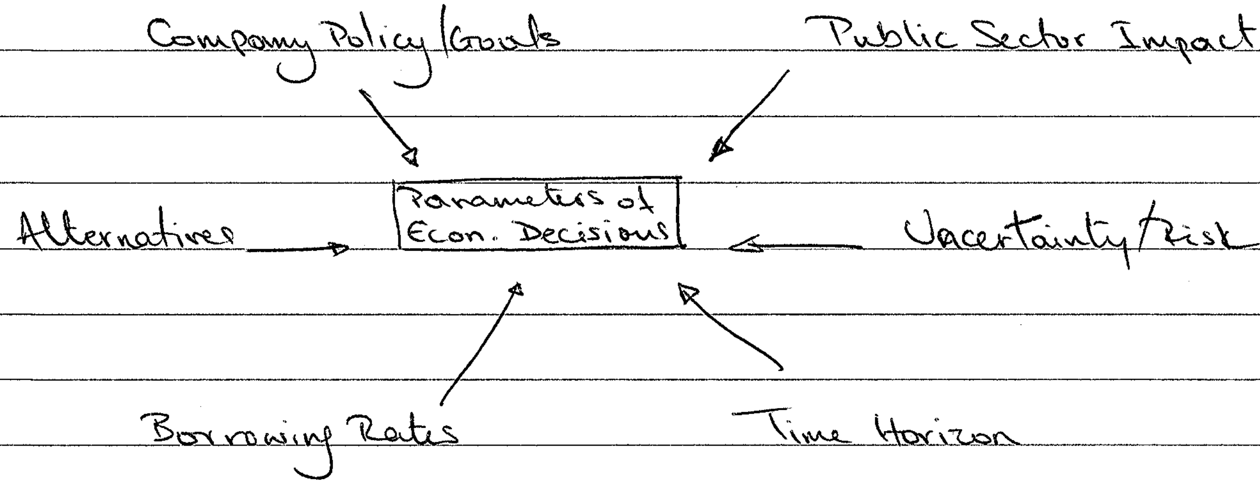
APPENDIX B

~~Chapter 3:~~

Economic Analysis of Alternatives

Parameters of Economic Decisions



~~Example: NYC~~

~~Econ~~

The object of the economic evaluation is to select the most cost-effective alternative that will satisfy the stated goals and objectives.

⇒ Goals & objectives clearly defined

& ~~define MARR > IRR~~ (for feasibility)

& define planning horizon (useful project live)

IRR > MARR

Time Value of money

I: interest rate per payment period

N: total number of periods

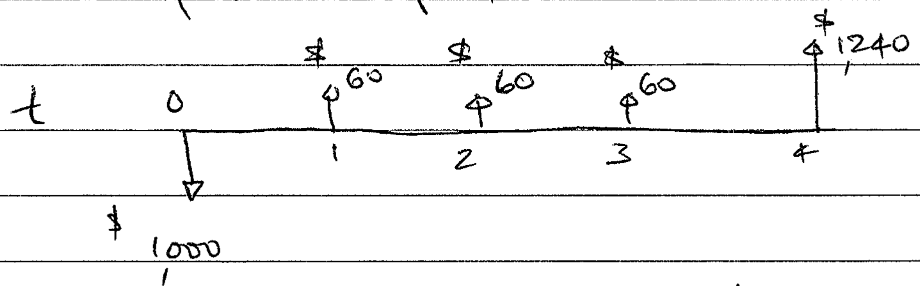
P: present value / present worth (time zero)

F: future value / future worth (time N)

A: annuity, or uniform payment occuring at uniform time

V_n: equivalent value at the end of specific period n

Cash flow concepts / conventions:



I = 6%

(a) ↑ arrow: positive cash flow (receipts, savings, etc)

↓ arrow: negative cash flow (payments, costs, etc.)

(b) interest ~~payments~~ occurs @ end of each time period

Interest ≡ the cost of money
(usually stated on an annual basis, APR)

Simple interest:

~~F = P(...)~~

$$F = P + P(ni) = P(1+ni)$$

Example:

\$18,000 loan for a car
for 4 years, @ 12% interest

$\Rightarrow$ Repayment $= \$18,000 + \$18,000(0.12)(4)$
$= \$18,000(1.48)$
$= \$26,640$

Payments: $\dfrac{\$26,640}{4 \text{ years}} = \$6,660/\text{year}$

✱ $= \$555/\text{month}$

Compounded interest

The interest is paid on the capital & interest unpaid balance
~~(only)~~

Derivation:

① Year	② Capital	③ Interest	④ = ② + ③ Future Worth
0	P	0	P
1	P	$P(i)$	$P(1+i)$
2	$P(1+i)$	$P(1+i)(i)$	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2(i)$	$P(1+i)^3$
⋮			
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1}\,i$	$P(1+i)^n$

(*)

Simple Interest (Example)

For what period of time will \$600 have to be invested to amount to \$1,725 if it earns 25% simple interest per year?

$$F = P + P(ni)$$

$$\Rightarrow 1725 = 600 + 600\left(n \times \frac{25}{100}\right)$$

$$\Rightarrow n = (1725 - 600)\left(\frac{1}{600 \times 0.25}\right) = 7.5 \text{ years}$$

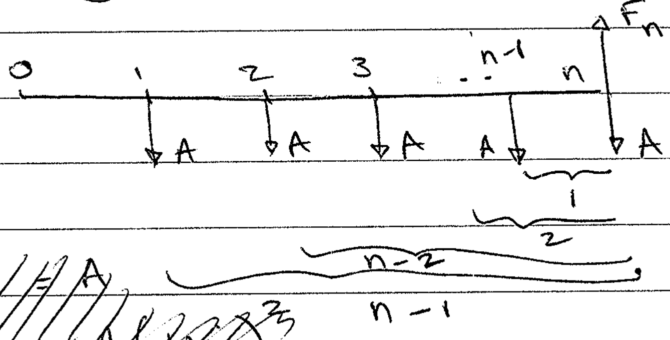
$$\Rightarrow \boxed{F_n = P\underbrace{(1+i)^n}_{\equiv \underbrace{(F/P,\, i,\, n)}_{\text{Single-payment Compound-amount factor}}}}$$

From above:

$$\boxed{P = \underbrace{\frac{1}{(1+i)^n}}_{(P/F,\, i,\, n)} F_n}$$

$(P/F, i, n)$ ↳ Single-payment present-worth factor

Equal Payment or Uniform Series Equation



In the future:

$$F_n = A + A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + \dots + A(1+i)^{n-2} + A(1+i)^{n-1}$$

$$\Rightarrow F_n = A\left[1 + (1+i)^1 + (1+i)^2 + \dots + (1+i)^{n-2} + (1+i)^{n-1}\right]$$

multiply by $(1+i)$

$$\Rightarrow (1+i) F_n = A\left[(1+i) + (1+i)^2 + (1+i)^3 + \dots + (1+i)^{n-1} + (1+i)^n\right]$$

Hence,

$$(1+i)F_n - F_n = A(1+i)^n - A$$

$$\Rightarrow \quad F_n\,(1+i-1) = A\left[(1+i)^n - 1\right]$$

$$\Rightarrow \quad \boxed{F_n = A \underbrace{\left[\frac{(1+i)^n - 1}{i}\right]}_{(F|A,\,i,\,n)}}$$

$(F|A, i, n) \equiv$ Equal-Payment-series compound-amount factor

From above:

$$\boxed{A = F_n \underbrace{\left[\frac{i}{(1+i)^n - 1}\right]}_{(A|F,\,i,\,n)}}$$

$(A|F, i, n) =$ Equal-Payment-Series Sinking-Fund factor

~~Also:~~ ~~Geometric Series (G)~~

From above

~~Note~~

equal-payment-series capital-recovery-factor

$$A = F_n\,(A|F, i, n)$$

$$= P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right] = P\underbrace{\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]}_{(A|P,\,i,\,n)}$$

$$\Rightarrow \quad P = A\underbrace{\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}_{(P|A,\,i,\,n)}$$

Equal-payment-series present-worth factor

Also, G: Geometric Series

~~Note: Solv: $F = P(F|P, i, n)$~~

~~$= A(A|P, i, n)(F|P, i, n)$~~

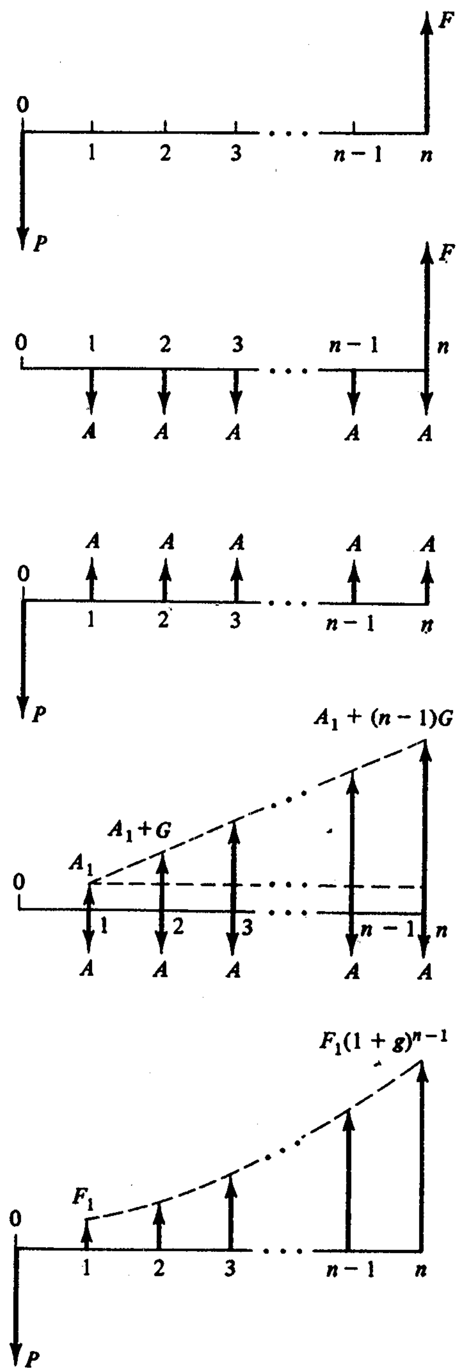
Note: Chain Rule applies

Ex. If Find P given F, then

$$P = F(P|F, i\%, n) = F\underbrace{(A|F, i\%, n)}_{A} * (P|A, i\%, n)$$

SCHEMATIC ILLUSTRATION OF THE USE OF INTEREST FACTORS

Find	Given	Formula
F	P	$F = P(^{F/P,i,n})$
P	F	$P = F(^{P/F,i,n})$
F	A	$F = A(^{F/A,i,n})$
A	F	$A = F(^{A/F,i,n})$
P	A	$P = A(^{P/A,i,n})$
A	P	$A = P(^{A/P,i,n})$
A	G	$A = A_1 + G(^{A/G,i,n})$
P	F_1	$P = F_1(^{P/A,g',n})/(1+g)$



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COMPOUND INTEREST FORMULAS

Definitions:

$i =$	effective interest rate per interest period, sometimes referred to as the discount rate or minimum attractive rate of return (MARR); given as a decimal number in the formulas below (i.e. 12% is equivalent to 0.12)
$n =$	number of compounding periods
$P =$	present sum of money (equivalent worth of one or more cash flows at a point in time called the present)
$F =$	future sum of money (equivalent worth of one or more cash flows at a point in time called the future)
$A =$	end-of-period cash flow (or equivalent end-of-period value) in a uniform series continuing for n periods (sometimes called "annuity")
$G =$	uniform increase or decrease in end-of-period cash flows or amounts (the arithmetic gradient)

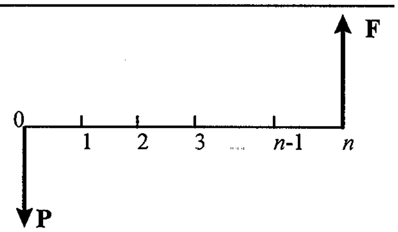
Compound interest formulas:

Single-payment compound-amount factor

$$(F/P,i\%,n) = (1+i)^n$$

Single-payment present worth factor

$$(P/F,i\%,n) = \frac{1}{(1+i)^n} = \frac{1}{(F/P,i\%,n)}$$

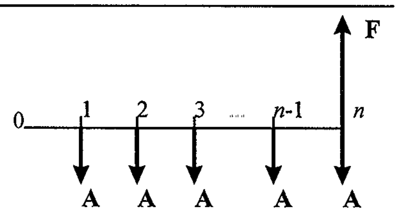


Uniform series compound amount factor

$$(F/A,i\%,n) = \frac{(1+i)^n - 1}{i}$$

Uniform series sinking amount factor

$$(A/F,i\%,n) = \frac{i}{(1+i)^n - 1} = \frac{1}{(F/A,i\%,n)}$$

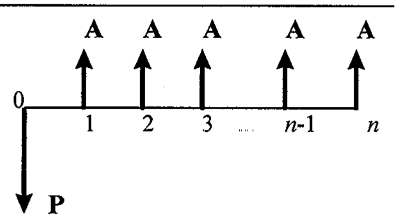


Uniform series present worth factor

$$(P/A,i\%,n) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Uniform series capital recovery factor

$$(A/P,i\%,n) = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{1}{(P/A,i\%,n)}$$



Arithmetic gradient present worth factor

$$(P/G,i\%,n) = \frac{(1+i)^n - in - 1}{i^2(1+i)^n}$$

Arithmetic gradient uniform series factor

$$(A/G,i\%,n) = \frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$