Example:

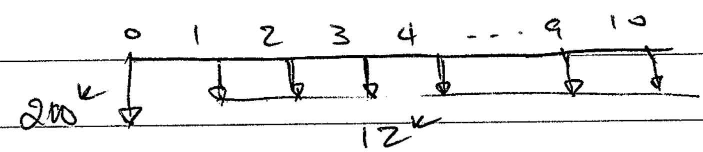
Bids from 2 HVAC contractors on AC systems

	Bid A	Bid B
Initial Cost	\$200,000	\$240,000
Annual operating Costs	\$12,000	\$8,000
Useful life	10 years	10 years

$i = 5\%$

Draw cash flows & compare bids based on a comparison of total present value costs.

Alt A:



$$PV_A = -200{,}000 - 12000(P|F, 5\%, 1) - 12{,}000(P|F, 5\%, 2) - \cdots - 12{,}000(P|F, 5\%, 10)$$

or

$$PV_A = -200{,}000 - 12{,}000(P|A, i, 10)$$

$$= -200{,}000 - 12{,}000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= -200{,}000 - 12{,}000(7.6743)$$

$$= -\$292{,}092$$

Similarly:

$$PV_B = -240{,}000 - 8{,}000(7.6743)$$

= ~~\$292,092~~

$$= -\$301{,}395$$

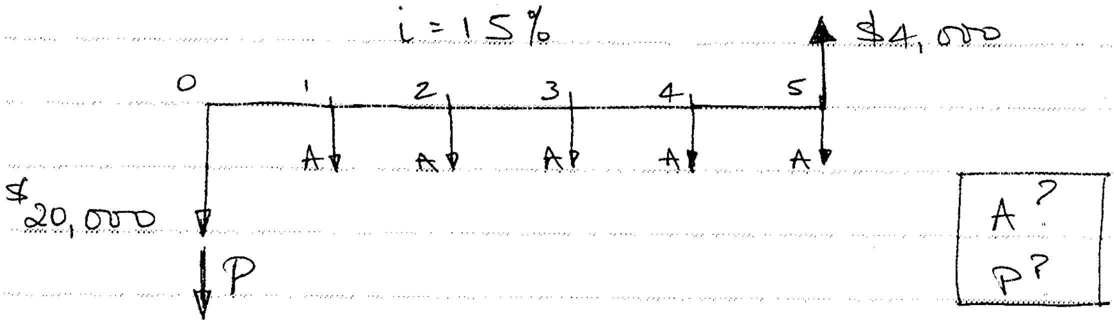
$PV_A > PV_B$

$\Rightarrow$ Choose "A"

PRESENT/FUTURE WORTH & UNIFORM / GRADIENT SERIES (EXAMPLES)

- Example 1:
  - Suppose that a $20,000 piece of equipment is expected to last 5 years and then result in a $4,000 salvage value. If the minimum attractive rate of return (interest rate) is 15%, what are the following values?
    - Annual equivalent (cost)
    - Present equivalent (cost)
    -
- Example 2:
  - Suppose that the equipment in example 1 is expected to be replaced 3 times with identical equipment, making 4 life cycles of 5 years each. To compare this investment correctly with another alternative that can serve 20 years, what are the following values when MARR=15%?
    - Annual equivalent (cost)
    - Present equivalent (cost)

EXAMPLE 1: SOLUTION



(a) $A = -20{,}000\,(A|P, 15\%, 5) + 4000\,(A|P, 15\%, 5)$

$= -20{,}000\,(0.2983) + 4{,}000\,(0.1483)$

$= -\$5{,}373$

(b) $P = -20{,}000 + 4{,}000\,(P|F, 15\%, 5)$

$= -20{,}000 + 4{,}000\,(0.4972)$

$= -\$18{,}011$

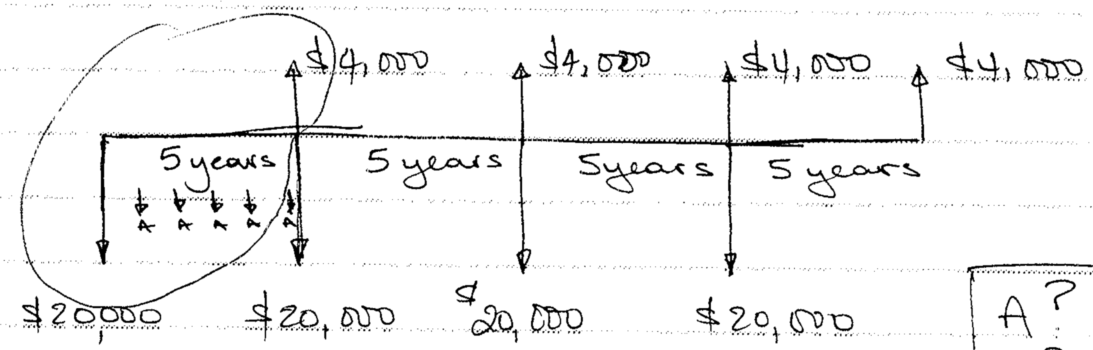
Alternatively:

$P = A\,(P|A, 15\%, 5)$

$= -5{,}373\,(3.3522)$

$= -\$18{,}011$

EXAMPLE 2: SOLUTION



A?
P?

(P|F for each 5-year cycle)

(a) $A = \left[-20000 + 4{,}000\,(P|F, 15\%, 5)\right](A|P, 15\%, 5)$

$= \left[-20{,}000 + 4{,}000(0.4972)\right](0.2983)$

$= -\$5.373$ (*)

(b) $P = -5{,}373\,(P|A, 15\%, 20)$

$= -5{,}373\,(6.2593)$

$= -\$33{,}629$

(*) NOTE:

Recognize that if the cash flows repeat each cycle, the annual equivalent for one cycle will be the same for all other cycles

Example:

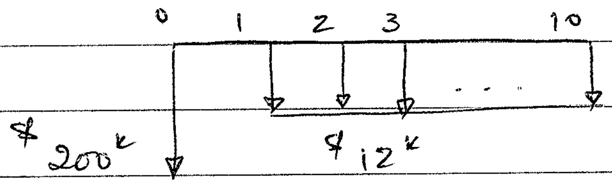
Bids from 2 HVAC Contractors on AC Units

~~Bid~~	Bid A	Bid B
Initial Cost	$200,000	$240,000
Annual Oper. Costs	$12,000	$8,000
Useful Life	10 years	12 years

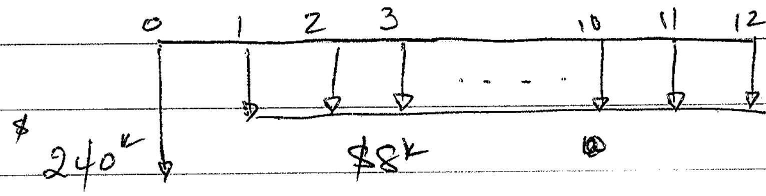
Draw Cash Flows & Compare bids based on comparison of uniform payments costs (EC)

$i = 5\%$

Alt. A



Alt B



~~Alt~~ 1: ~~AC(A/P, i, n)~~

~~EC = $200k + $12k~~

~~EC = A (t=0) + A~~

$$EC_A = -\$200^k (A|P, 5\%, 10) - \$12^k$$
$$= -200{,}000 \left[\frac{(1+i)^n i}{(1+i)^n - 1}\right] - 12{,}000$$
$$= -200{,}000 \left[\frac{(1.05)^{10}(0.05)}{(1.05)^{10} - 1}\right] - 12{,}000$$
$$= -200{,}000 (0.1295) - 12{,}000$$
$$= -25{,}901 - 12{,}000$$
$$= -\$37{,}901$$

$$EC_B = -\$240^k (A|P, 5\%, 12) - 8{,}000$$
$$= -240{,}000 \left[\frac{(1.05)^{12}(0.05)}{(1.05)^{12} - 1}\right] - 8{,}000$$
$$= -240000 (0.1128) - 8{,}000$$
$$= -27{,}078 - 8{,}000$$
$$= -\$35{,}078$$

$\Rightarrow$ Alt B is preferred

NOTE:

Equivalent Annual Method (EAV) was used because of unequal useful ~~life~~ lives of alternatives

If comparison based on P, then Net Present Value Analysis (NPV)

INTEREST COMPOUNDED OTHER THAN YEARLY

r: annual percentage rate (APR)

i_{per}: periodic interest rate

i_a: annual effective interest rate

m: compounding frequency (i.e. number of interest periods per year)

~~n: number of interest periods per payment period~~

k: number of ~~interest~~ payment periods per year

Step 1: Relate the annual effective interest rate to the APR

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

Step 2: Convert the annual effective interest rate & convert it to equivalent interest for the period of interest.

$$i_{per} = (1 + i_a)^{1/k} - 1$$

Example:

To obtain a home mortgage for \$135,000 for 30 years, at 8.5% APR compounded daily. Determine monthly payments and the amount of interest paid over the life cycle of the ~~project~~ loan.

(a) Determine i_a

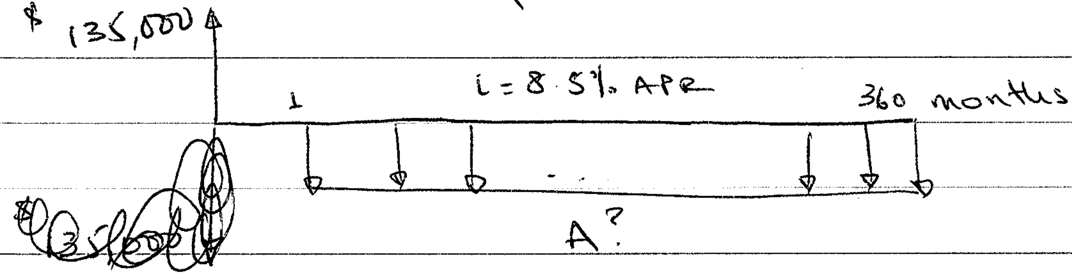
$$i_a = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.085}{365}\right)^{365} - 1 = 0.0887$$

(b) Determine i_{per}:

$$i_{per} = (1 + i_a)^{1/12} - 1 = (1 + 0.0887)^{1/12} - 1 = 0.007108$$

$$= 0.7108\% \text{ per month}$$

(c) Draw the cash flow



(d) Determine A (monthly payments)

$$A = P(A|P, i, n)$$

$$= (135{,}000)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= (\$135{,}000)\left[\frac{0.007108(1.007108)^{360}}{1.007108^{360} - 1}\right] = \$1040.87/\text{month}$$

(e) ~~Total~~ Total amount paid

$$\left(\frac{\$1040.87}{\text{month}}\right)(360 \text{ months}) = \$374{,}712$$

(i.e. \$239,712 on interest)