SWIPT THROUGH EIGEN-DECOMPOSITION OF MIMO CHANNELS

(Invited paper)

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ABSTRACT

In this paper, we theoretically investigate a new technique for simultaneous wireless information and power transfer in multiple-input multiple-output (MIMO) point-to-point with radio frequency energy harvesting capabilities. The proposed technique exploits the spatial decomposition of the MIMO channel and uses the eigenchannels either to convey information or to transfer energy. An optimization problem that minimizes the total transmitted power subject to maximum power per eigenchannel, information and energy constraints is formulated as a mixed-integer nonlinear program and solved to optimality using mixed-integer second-order cone programming.

Index Terms — RF energy harvesting, SWIPT, MIMO channel, SVD, optimization, MISOCP.

1. INTRODUCTION

The integration of the energy harvesting concept into communication networks is a hot research topic with high socioeconomic impacts. Conventional approaches focus on natural energy harvesting sources such as solar power, wind, mechanical vibrations and refer to new protocols and transmission techniques that efficiently handle the harvested energy [1]. However, harvesting from natural sources is mainly an unpredictable and unstable process which could be critical for applications with strict quality-of-service (QoS) constraints. Recently there is a lot of interest to use electromagnetic radiation as a potential renewable energy resource [2]. The key idea of this concept is that electromagnetic waves convey energy, which can be converted to DC-voltage by using specific rectenna circuits [3, 4]. Since radio signals carry both information and energy at the same time, a unified study on simultaneous wireless information and power transfer (SWIPT) is an emergent topic.

Although information theoretic studies assume that the same signal can be used for both information and power transfer, this simultaneous transfer is not possible without losses due to practical limitations. In [5], the authors introduce three main practical techniques for simultaneous information and power transfer (SWIPT): a) “time switching” (TS), where the receiver switches in time between decoding information and harvesting energy and b) “power splitting” (PS), where the receiver splits the received radio-frequency (RF) signal in two parts for decoding information and harvesting energy, respectively, and c) “Antenna switching” (AS), where the receiver is equipped with multiple antennas and splits the antennas into two groups; one group is used for conventional information decoding (ID), and the second group for RF energy harvesting (EH).

In contrast to the conventional SWIPT approaches i.e., TS, PS, AS, we propose a new technique for SWIPT in the spatial domain for a basic point-to-point multiple-input multiple-output (MIMO) channel. In this work, spatial domain does not refer to the antenna elements [6, 7] but mainly on the spatial degrees of freedom of the channel. Based on the singular value decomposition (SVD) of the MIMO channel, the communication link is transformed to parallel channels that can convey either information or energy; this binary allocation is in respect to the current practical limitations. We study the minimization of the transmitted power when the receiver is characterized by maximum power per eigenchannel, information rate and power transfer constraints which is a mixed-integer nonlinear optimization problem and propose several solution methods. It is worth noting that the main purpose of this work is to introduce a new technique for SWIPT in MIMO systems; this technique is studied from a theoretical standpoint and practical implementation is beyond the scope of this paper. The proposed technique is extended to MIMO channels with channel estimation error in [8].

2. SYSTEM MODEL & PROBLEM FORMULATION

We assume a simple point-to-point MIMO model consisting of one source with $N_T$ transmit antennas and one destination with $N_R$ receive antennas. The source is connected to a constant power supply while the destination has RF transfer capabilities and can harvest energy from the received electromagnetic radiation. We consider a flat fading spatially uncorrelated Rayleigh MIMO channel where $H \in \mathbb{C}^{N_R \times N_T}$ denotes the channel matrix. The channel remains constant during one transmission time-slot and changes independently from one slot to the next. The entries of $H$ are assumed to be independent, zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance (which en-
where rank(\(H\)) = \(N = \min\{N_T, N_R\}\). The received signal is described by
\[
y = Hx + n, \quad (1)
\]
where \(x \in \mathbb{C}^{N_T \times 1}\) denotes the transmitted signal with \(\mathbb{E}[xx^{\dagger}] = Q\) and \(n \in \mathbb{C}^{N_R \times 1}\) represents the noise vector having ZMCSGC entries of unit variance. By using the SVD of the \(H\) channel, the capacity of the MIMO channel is
\[
I(x; y) = \sum_{i \in \mathcal{N}} \log (1 + P_i \lambda_i), \quad (2)
\]
where \(\lambda_i, i \in \mathcal{N} = \{1, ..., N\}\), is the \(i\)-th eigenvalue of \(H H^H\), and \(P_i\) is the power allocated to the \(i\)-th eigenchannel. It is also assumed that \(\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_N\). The MIMO channel is decomposed in \(N\) parallel SISO channels with
\[
\tilde{y}_i = \sigma \tilde{x}_i + \tilde{n}_i, \quad \text{with} \quad i \in \mathcal{N}, \quad (3)
\]
where \(\tilde{n}_i\) denotes the AWGN for the \(i\)-th parallel channel and has the same distribution with \(n_i\) (due to the unitary transformation) in reception. We assume that the destination is characterized by both information rate and RF EH requirements; this means that for the duration of one transmission the destination requires an information rate \(C_I\) and energy \(C_{EH}\).

### 2.1. SWIPT optimization problem

The proposed scheme exploits the SVD structure of the MIMO channel and achieves SWIPT in the spatial domain. More specifically, the transformation of the MIMO channel to \(N\) parallel SISO channels allows the simultaneous transfer of data traffic and RF energy by using an eigenchannel either to convey information or energy. An eigenchannel cannot be used to convey both information and energy; this limitation refers to practical constraints and is inline with the other approaches proposed in the literature, e.g., power splitting. During each transmission an appropriate optimization problem is solved, which determines the usage of each antenna and a switching mechanism selects the appropriate circuits for ID or EH.

In this paper, we focus on the joint eigenchannel assignment (into ID or EH) and power allocation problem under both information and energy constraints. In particular, we consider the following optimization problem
\[
\begin{align*}
\min P &= \sum_{i \in \mathcal{N}} P_i, \quad (4a) \\
\text{s.t.} \quad &\sum_{i \in \mathcal{N}} \log (1 + P_i \lambda_i) \geq C_I, \quad (4b) \\
&\sum_{i \in \mathcal{N}} (1 - \pi_i) P_i \lambda_i \geq C_{EH}, \quad (4c) \\
&0 \leq P_i \leq P_{\max}, \quad \pi_i \in \{0, 1\}, \quad i \in \mathcal{N}, \quad (4d)
\end{align*}
\]
where \(P_{\max}\) indicates the maximum power that can be used in each eigenchannel, while binary variable \(\pi_i\) indicates whether the \(i\)-th eigenchannel is used for ID (\(\pi_i = 1\)) or EH (\(\pi_i = 0\)). Note that terms \(\log((1 + P_i \lambda_i))\) are equivalent to the more intuitive representation \(\pi_i \log(1 + P_i \lambda_i)\) when \(\pi_i \in \{0, 1\}, \quad i \in \mathcal{N}\).

### 3. OPTIMAL POWER ALLOCATION WITH FIXED EIGENCHANNEL ASSIGNMENT

In this section, we develop a waterfilling-like procedure to solve the optimal power allocation problem for a given eigenchannel assignment, which is essential for the development of a low complexity algorithm for problem (4).

Let, \(\mathcal{T}\) and \(\mathcal{E}\), denote the sets of eigenchannels assigned for information and EH respectively; then, the optimal power allocation problem can be defined as:
\[
\begin{align*}
\min \sum_{i \in \mathcal{T}} P_i + \sum_{i \in \mathcal{E}} P_i, \quad (5a) \\
\text{s.t.} \quad &\sum_{i \in \mathcal{T}} \log (1 + P_i \lambda_i) \geq C_I, \quad (5b) \\
&\sum_{i \in \mathcal{E}} P_i \lambda_i \geq C_{EH}, \quad (5c) \\
&0 \leq P_i \leq P_{\max}, \quad i \in \{\mathcal{T} \cup \mathcal{E}\}. \quad (5d)
\end{align*}
\]

Notice that the above problem is decomposable to the information subproblem with objective \(\min \sum_{i \in \mathcal{T}} P_i\) and constraints (5b), (5d), and the EH subproblem with objective \(\min \sum_{i \in \mathcal{E}} P_i\) and constraints (5c), (5d).

The EH subproblem can be optimally solved by allocating as much power as needed to the “better” eigenchannels, i.e., the ones with higher \(\lambda_i\), until constraint (5c) is satisfied, or the problem is deemed infeasible. In mathematical terms the solution of the EH subproblem for \(k = 1, ..., |\mathcal{E}|\) is
\[
P_{\epsilon_k} = \max \left\{0, \min \left\{C_{EH} - P_{\max} \sum_{j=1}^{k-1} \lambda_{\epsilon_j}, P_{\max}\right\}\right\}, \quad (6)
\]
where index \(\epsilon_k\) denotes the EH eigenchannel with the \(k\)-th strongest eigenchannel in \(\mathcal{E}\).

The solution of the information subproblem is summarized in the following theorem.

**Theorem 1.** The solution of the information subproblem, if the problem is feasible (i.e., \(\sum_{i \in \mathcal{T}} \log(1 + P_{\max} \lambda_i) \geq C_I\)), is given by:
\[
P_i^* = \begin{cases} 
0, & \nu < 1/\lambda_i \\
\nu - 1/\lambda_i, & 1/\lambda_i \leq \nu \leq 1/\lambda_i + P_{\max} \\
P_{\max}, & \text{otherwise},
\end{cases}
\]

where \(\nu\) is the solution of the information subproblem.
Algorithm 1: Optimal solution to information subproblem

1. **Init.** Set $q = \text{sortAscending}([1 \otimes \lambda, 1 \otimes \lambda + P_{\text{max}}])$. Set $\rho = 2^{C_{ij}}; r = 0$;

2. **Check problem feasibility:** $\sum_{i \in \mathcal{I}} (\log(1 + P_{\text{max}} \lambda_i)) \geq C_i$; if problem is infeasible stop, else continue.

3. **for** $j = 1, ..., |\mathcal{I}|$ **do**

4. **if** $(q_j = 1/\lambda_i, i \in \mathcal{I})$ **then**

5. Set $r = r + 1; \rho = \rho q_j; \nu = \rho^{1/r}; \mathcal{I}_1 = \mathcal{I}_1 \cup \{i\}$;

6. **else if** $(q_j = 1/\lambda_i + P_{\text{max}}, i \in \mathcal{I})$ **then**

7. Set $\mathcal{I}_1 = \mathcal{I}_1 \cup \{i\}; r = r - 1$;

8. Set $\rho = \rho / q_j; \nu = \rho^{1/r}; \mathcal{I}_2 = \mathcal{I}_2 \cup \{i\}$;

9. **if** $\nu \in [q_j, q_{j+1}]$ **then**

10. Set $\nu = \nu_c$; break;

11. Set $P^*_i = \min \{P_{\text{max}}, \max \{0, \nu - 1/\lambda_i\} \}, i \in \mathcal{I}$;

where the Lagrange multiplier $\nu$ is derived from:

$$\nu = \rho^{(1/|\mathcal{I}_i|)}, \quad \rho = 2^{C_i} \prod_{i \in \mathcal{I}_1} (1/\lambda_i)$$

(8)

Sets $\mathcal{I}_1$ and $\mathcal{I}_2$ are defined as $\mathcal{I}_1 = \{i : 0 < P^*_i < P_{\text{max}}\}$ and $\mathcal{I}_2 = \{i : P^*_i = P_{\text{max}}\}$.

**Proof:** The proof is similar to the proof of the waterfilling theorem with maximum power constraints and is omitted.

Theorem 1 indicates that the computation of the optimal power allocation can be easily obtained from (7), provided that the value of $\nu$ satisfying (8) is found. This can be achieved if the eigenchannels in sets $\mathcal{I}_1$ and $\mathcal{I}_2$ are found. Towards this direction, first we need to sort the $2|\mathcal{I}|$ values $1/\lambda_i$ and $1/\lambda_i + P_{\text{max}}$ in ascending order; let us assume that this order is $q_1 \leq q_2 \leq \ldots \leq q_{2|\mathcal{I}|}$. If $\nu \in [q_j, q_{j+1}]$, sets $\mathcal{I}_1$ and $\mathcal{I}_2$ can be constructed and a candidate value, $\nu_c$, can be computed from (8); in case $\nu \in [q_j, q_{j+1}]$ then it is optimal, otherwise the subsequent range of values needs to be examined, i.e., $[q_{j+1}, q_{j+2}]$. For example, if $|\mathcal{I}| = 2$, $1/\lambda_1 = 1, 1/\lambda_2 = 2, P_{\text{max}} = 0.5$, and $\nu = 2.3$, with $q_i = \{1, 1.5, 2, 2.5\}$, then, according to (8), it will be true that $\mathcal{I}_1 = \{2\}$ and $\mathcal{I}_2 = \{1\}$, as $\nu - 1/\lambda_1 > P_{\text{max}}$ and $0 < \nu - 1/\lambda_2 < P_{\text{max}}$.

A computationally efficient method to obtain the optimal value of $\nu$ in $O(|\mathcal{I}|)$ is provided in Algorithm 1, where $\otimes$ denotes element-wise division. The idea is to sequentially keep updating $\rho$ and $\nu = |\mathcal{I}_i|$ for cheap computation of $\nu_c$, while examining new regions. In Algorithm 1, lines 4-5 describe the addition of eigenchannel $i$ into set $\mathcal{I}_1$ (when $q_j = 1/\lambda_i$), while lines 6-8 the removal of eigenchannel $i$ from set $\mathcal{I}_1$ and its addition into set $\mathcal{I}_2$ (when $q_j = 1/\lambda_i + P_{\text{max}}$).

4. **JOINT EIGENCHANNEL ASSIGNMENT AND POWER ALLOCATION**

Problem (4) is nonlinear and combinatorial in nature, and hence very hard to solve. In this section a Mixed-Integer Second Order Cone Programming (MISOPC) formulation is derived that allows its optimal solution with standard optimization solvers. In addition, we provide a polynomial algorithm that optimally solves the problem for the special case of $P_{\text{max}} = \infty$.

4.1. **Optimal MISOPC solution**

Towards the optimal solution of (4), we write it as:

$$\min \sum_{i \in \mathcal{N}} P_i$$

(9a)

s.t. $\prod_{i \in \mathcal{N}} (1 + P^*_i \lambda_i) \geq 2^{C_i}$

(9b)

$$\sum_{i \in \mathcal{N}} P_i \lambda_i - P^*_i \lambda_i \geq C_{EH}$$

(9c)

$$0 \leq P^*_i \leq P_i \leq P_{\text{max}}, \pi_i \in \{0, 1\}, i \in \mathcal{N}.$$ (9d)

$$P^*_i = \pi_i P_i, i \in \mathcal{N}.$$ (9e)

Formulation (9) is not MISOPC due to the nonlinear equalities (9e) and the general product of terms in (9b). Inequalities (9e) are equivalent to $0 \leq P^*_i \leq P_i$, combined with the following Mixed Integer Linear Programming expressions

$$P^*_i \leq \pi_i P^*_i, P^*_i \geq P_i - (1 - \pi_i) P_{\text{max}}, i \in \mathcal{N}.$$ (10)

Next, we will show that the general product appearing in (9b) can be represented by a hierarchy of convex second-order rotated cone (SORC) constraints of the form $x_i x_j \geq x^2_0, x_0 \geq 0, x_2 \geq 0$ using proposition 1.

**Proposition 1.** ([9, p.105]): The geometric mean constraint (GM) $x_1 ... x_{2^k} \geq t^{2^k}, t > 0, i = 1, ..., 2^k$ is convex and can be represented by a hierarchy of SORC constraints. Let us define existing variables as $x_i \equiv x_{0i}$ and new variables $x_{k,i} \geq 0, k = 1, ..., l, i = 1, ..., 2^{k-l}$. Then, the GMC is equivalent to the following SORC constraints

$$k: x_{k-1,2^k-1} \geq x^2_{1,k}, k = 1, ..., 2^k-1, k = 1, l, x_{1,1} \geq t.$$ (11a)

To bring (9b) into the GMC form, we define $M = \lceil \log N \rceil, N_0 = 2^M$, and set $x_{0i} = 1 + P^*_i \lambda_i \geq 0, i \in \mathcal{N}, x_{0i} = 1, i = N + 1, ..., N_0$ and $t = 2^{G_{1i}}/N_0$, with $l = M$. Towards this direction, a total of $N_0$ new variables are needed, $x_{k,i} \geq 0, k = 1, ..., l, i = 1, ..., 2^M-k$, and $N_0/2$ SORC constraints. Hence, problem (4) can be rewritten as:

$$\min \sum_{i \in \mathcal{N}} P_i$$

(11a)

s.t. $x_{k-1,2^k-1} \geq x^2_{1,k}, k = 1, ..., M, i = 1, ..., 2^{M-k}$,

(11b)

$$x_{M,i} \geq t, t = 2^{G_{1i}}/N_0,$$ (11c)

$$x_{0,i} = 1 + P^*_i \lambda_i, i \in \mathcal{N},$$ (11d)

$$x_{0,i} = 1, i = N + 1, ..., N_0,$$ (11e)

$$x_{k,i} \geq 0, k = 0, ..., M, i = 1, ..., 2^{M-k},$$ (11f)

Constraints (9c), (9d) and (10). (11g)
Remark 2. Eigenchannels with non-zero transmitted power in each eigenchannel, so that only one EH eigenchannel has nonzero power. When \( P_{\text{max}} \) is potentially optimal assignment in Algorithm 2. The solution of each assignment can be easily obtained and vice versa. Hence, a total of \( N \) different assignments combinations are examined resulting in computational complexity \( O(N^2) \).

GAE heuristic is outlined in Algorithm 3. First, Algorithm 2 is examined as a way to optimally solve cases of low maximum transmitted power, i.e., with \( P_{\text{max}} \leq \sum P_i \), \( i \in \mathcal{N} \). If that is not the case, GEA implements the main idea of assigning ID and EH eigenchannels in groups.

6. NUMERICAL RESULTS

Computer simulations are carried-out in order to evaluate the performance of the MISOCSP and GEA algorithms. Since the scope of the paper is to highlight the theoretical idea of SWIPT in point-to-point MIMO setup, only small-scale fading is considered for the wireless medium. We consider a 8 × 8 MIMO system with \( P_{\text{max}} = 2 \) and perfect harvesting efficiency. For each different experimental configuration, the results are averaged over 1000 randomly generated problem instances, in which the entries of the channel matrix are independent and identically distributed ZMCSGC random variables with unit variance. Mathematical modeling and solution of MISOCSP formulation is done using Gurobi [11].

Fig. 1 demonstrates the effect of the ID and EH thresholds on the total transmitted power, when the MISOCSP solution is invoked. The figure indicates that by increasing the ID or EH thresholds the power increases as well. For the ID threshold, there is a significant nonlinear power increase due to the logarithmic relationship between power and information rate. Interestingly, the effect of the EH threshold is small which indicates that for the specific configuration the ID threshold dominates the total power increase.

Fig. 2 illustrates the relative percentage optimality deviation (RPD) of the GEA solution compared to the optimal MISOCSP solution. GEA achieves average performance within 10% from the optimal in all scenarios examined except for \( C_I = 18 \) - \( C_{EH} = 10 \). An interesting observation is that there is no clear conclusion regarding the effect of the ID and EH thresholds on RPD. For example, while there is a tendency for the RPD to grow as a function of \( C_I \), for \( C_{EH} = 1 \) RPD is minimal (less than 1%) for a range of values \( C_I \in (6, 14) \). Note that the absence of some bars in-
Fig. 1. Effect of the EH and ID thresholds on the total transmitted power.

Fig. 2. Relative percentage deviation of GEA from the optimal MISOCP solution.

dicates 0% deviation of GEA from optimality, i.e., optimal performance.

7. CONCLUSIONS

In this paper, we have theoretically investigated SWIPT in the spatial domain for a MIMO channel with RF EH capabilities. By using SVD decomposition for the wireless channel under perfect channel knowledge, the proposed technique uses the eigenchannels to convey either information or energy, with the goal of minimizing the overall transmitted power subject to information and energy constraints. Although the examined problem is non-linear and combinatorial in nature, an MISOCP formulation have been developed that provides optimal solutions. A polynomial-complexity optimal algorithm has been developed for the special case of having no restric-


tions on the maximum power allowed in each eigenchannel. We have also shown that for known eigenchannel assignment the power allocation problem can be addressed in a waterfilling fashion. Finally, a low-complexity algorithm is presented which produces near-optimal solutions for a wide range of parameter configurations.

REFERENCES


