Towards Distributed Online Cooperative Traffic Signal Control using the Cell Transmission Model

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Abstract—Traffic signal control is a key ingredient in intelligent transportation systems (ITS) to increase the capacity of existing urban transportation infrastructure. However, to achieve optimal system-wide operation it is essential to coordinate traffic signals at various intersections. In this paper we model the multiple-intersections traffic signal control problem using the cell transmission model. For its solution, we propose two online distributed strategies, which are based on spatially and temporally decomposing the problem into subproblems associated with different intersections and iteratively solving them by exchanging information between neighboring intersections. Simulation results for a four intersection topology indicate that the proposed strategies achieve distributed, online and close to optimal signal timing plans.

Keywords: intelligent transportation systems (ITS), traffic signal control, cell-transmission model, online, distributed, mixed-integer linear programming.

I. INTRODUCTION

Traffic signal control can bring substantial reduction to traffic congestion leading to improved conditions both for the drivers (better travel times and convenience) and the environment (reduced air pollution and energy consumption). Furthermore, recent advancements in electronics, sensing, and ICT (information and communication technology) allow the real-time collection and processing of traffic data, as well as the deployment of intelligent controllers for the efficient operation of a transportation system. However, controlling the traffic signals of a transportation network constitutes a significant challenge due to the large-scale nature and complexity of the problem, the uncertain and dynamic behavior of the network (e.g. weather, accidents, events) and the patterns of different driver behaviors. Hence, several different solution approaches have been proposed.

The majority of techniques consider the single intersection problem neglecting interrelation effects with other intersections. These approaches aim to optimize some measure-of-interest (mean delay, throughput) based on the state of the intersection. There is a wealth of techniques employed for its solution such as mathematical programming [1], stochastic control [2], as well as computational and artificial intelligence [3]. However, by considering intersections atomically the offset between intersections is not optimized leading to frequent vehicle stops. Also measures-of-interest are optimized locally instead of globally and may lead to poor global performance.

Fixed or pre-timed signal control strategies optimize signal timing plans based on historical data so that a fixed signal timing plan is in place for different periods of the day. These approaches usually rely on the development of MILP formulations using the Cell Transmission Model (CTM) [4], [5]. However, because the formulated problems are usually NP-hard, meta-heuristic techniques such as genetic algorithms [3] are often employed to achieve close to optimal and timely solutions. Pre-timed signal control strategies can perform fairly well during peak traffic periods, but their performance deteriorates during off-peak periods or when unexpected events create unanticipated traffic conditions (e.g. accidents).

Distributed online algorithms dynamically adjust the split plans of multiple intersections based on the network state. Due to the complex and large-scale nature of the problem these algorithms employ simplistic system models that lead to low-complexity but inaccurate techniques. Techniques in this area usually draw inspiration from artificial and computational intelligence techniques such as reinforcement learning (RL) [6] and Markov decision processes [7], or control approaches such as back-pressure routing [8].

In this paper cooperative distributed online algorithms for traffic signal control based on the cell transmission model are proposed. The algorithms are based on spatially and temporally decomposing the problem and iteratively solving the produced subproblems by individual intersection controllers. Spatial decomposition is achieved by dividing the considered transportation topology into single-intersection areas. Temporal decomposition is achieved by separating the time-horizon considered into small time-windows (10-15 mins). Because the subproblem solution presumes information associated with neighboring intersections, such as incoming flows and signal plans, intersection controllers cooperate with each other by exchanging appropriate data; this results in the distributed solution of the global problem. The Local Distributed Algorithm (LDA) considers only incoming flow information from neighboring areas to optimize the signal timing plan of each intersection, while the neighborhood distributed algorithm (NDA) also considers the signal timing plans from neighboring intersections allowing better offsets, cycle lengths and split plans. Contrary to other distributed algorithms that myopically control traffic signals based on the current network state, our approach optimizes the problem for a look-ahead time-window achieving close to optimal results.

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II. CELL TRANSMISSION MODEL

CTM [9] is a discrete analog of the well-known first-order Lighthill-Whitham-Richards (LWR) continuum flow model [10] which is based on the fundamental relationship for the conservation of flow and supplemented by the assumption that traffic flow at road point \( x \) at time \( t \), \( q(x, t) \), is only a function of traffic density \( k(x, t) \). The form of this flow-density function is specified using a flow-density model (e.g. Greenshields, Pipes, or Van Aerde models) and calibrated by estimating macroscopic traffic flow parameters (such as the free-flow speed, jam-density, maximum flow or capacity and speed-at-capacity) for a specific road configuration.

In the CTM both time and space are discrete. Each road segment is divided into homogeneous sections called cells, while time is partitioned in a way that one vehicle takes one time-unit to travel through one cell at free-flow speed. When CTM assumes a piecewise-linear flow-density relationship, the LWR model is simplified to the difference equations (1)-(3) which constitute the fundamental relationships of CTM.

\[
\begin{align*}
    z_{i,t} & = \min(n_{i,t}, Q_{i,t}, Q_{i+1,t}, W(N_{i+1,t} - n_{i+1,t})) \\
    y_{i+1,t} & = z_{i,t} \\
    n_{i,t+1} & = n_{i,t} - z_{i,t} + y_{i,t}
\end{align*}
\]

In the above equations, \( z_{i,t} \), \( y_{i+1,t} \), and \( n_{i,t} \) represent the number of vehicles leaving cell \( i \), entering cell \( i+1 \) and are inside cell \( i \) at time \( [t, t+1) \) respectively. \( Q_{i,t} \) represents the maximum number of vehicles that are allowed to leave cell \( i \) at time \( t \), while \( N_{i,t} \) denotes the maximum number of vehicles that are allowed to reside into cell \( i \) at time \( t \). \( W \) is the ratio between the shock-wave propagation speed and the free-flow speed and indicates how fast a vehicle queue is formed.

III. CENTRALIZED PROBLEM FORMULATION

This section details the centralized mathematical formulation for the multiple intersection traffic signal control (MITSC) problem which involves the optimal selection of split plans, cycle lengths and offsets of multiple traffic signals over a time-horizon \( T \). Optimization is performed for some measure-of-interest such as mean/total vehicle delay, stoppage time and throughput. Traffic dynamics are incorporated into the optimization problem through CTM described in section II. Our formulation builds on existing centralized approaches such as [4] and [5], by relaxing the assumption of an initially zero-state system; this allows the temporal decomposition of the original problem into subproblems spanning relatively small time-windows \( T_w \) by allowing the optimization of initially non-zero state networks. Before arriving at the final MITSC formulation, the constraints of the problem are introduced.

A. CTM constraints

To incorporate the CTM-based traffic flows into a mathematical programming formulation, its fundamental equations (1)-(3) must be converted into a suitable LP form for all cell types considered. Without loss of generality, we assume that the transportation network is homogeneous so that \( N_{i,t} = N \) and \( Q_{i,t} = Q \), \( i \in A \), \( \forall t \), where \( A \) denotes the set of all cells. Here, four different cell types are discussed: ordinary, origin, destination and intersection.

Ordinary cells: Ordinary cells have both inflow and outflow of vehicles as well as non-zero capacity at all times. Hence, the CTM equations are similar to Eqs. (1)-(3):

\[
\begin{align*}
    y_{i,t} & = \min(n_{i,t}, W(N - n_{i+1,t}), Q), \ i \in \mathcal{E}, \ \forall t \quad (4) \\
    n_{i,t+1} & = n_{i,t} + y_{i-1,t} - y_{i,t} \quad (5)
\end{align*}
\]

Notice that although eq. (5) is linear, the presence of the \( \min \) function makes Eq. (4) nonlinear. However, it can be transformed into the following equivalent LP form

\[
\begin{align*}
    \max \ y_{i,t} \\
    y_{i,t} & \leq W(N - n_{i+1,t}), \ i \in \mathcal{E}, \ \forall t \quad (6) \\
    y_{i,t} & \leq n_{i,t}, \ 0 \leq y_{i,t} \leq Q,
\end{align*}
\]

where \( \mathcal{E} \) denotes the set of ordinary cells.

Origin cells: Origin cells are similar to ordinary cells but instead of receiving inflow traffic from other cells they receive exogenous inflow traffic \( D_{i,t} \). Hence, Eq. (4) still holds and the LP program of (6) is used, while Eq. (5) becomes

\[
\begin{align*}
    n_{i,t+1} & = n_{i,t} + D_{i,t} - y_{i,t}, \ i \in \mathcal{O}, \ \forall t
\end{align*}
\]

where \( \mathcal{O} \) denotes the set of origin cells.

Destination cells: These cells send their outflow traffic outside the network without restriction on capacity and with infinite space at their destination. Hence, Eqs. (4) and (5) are reduced to

\[
\begin{align*}
    y_{i,t} & = n_{i,t} \\
    n_{i,t+1} & = n_{i-1,t}, \ i \in \mathcal{D}, \ \forall t
\end{align*}
\]

where \( \mathcal{D} \) denotes the set of destination cells.

Intersection cells: Intersection cells are ordinary cells with the exception that their capacity \( q_{i,t} \) depends on the state of the traffic signals. If the traffic signal for cell \( i \) is green at time \( t \), then \( q_{i,t} = Q \), otherwise it is equal to zero. To achieve this restriction we introduce binary variables \( w_t \) which take the value 1 if the particular signal phase is green and 0 otherwise. For one-way roads, each traffic signal controlled intersection can be modeled by two intersection cells \( (i, j) \), \( i \in \mathcal{I}_1, j \in \mathcal{I}_2 \) and \( (i, j) \in \mathcal{I} \), as well as one binary variable for each time step, \( w_t \). \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) denote the sets of the first and second cells comprising an intersection, while \( \mathcal{I} \) denotes the set of cell pairs indicating different intersections. In this case, the variable capacities \( q_{k,t} \) of an intersection cell \( k = \{i, j\} \), \( (i, j) \in \mathcal{I} \) are given by

\[
q_{k,t} = \begin{cases} 
    w_t Q, & k = i \in \mathcal{I}_1 \\
    (1 - w_t) Q, & i \in \mathcal{I}_1, k \in \mathcal{I}_2, (i, k) \in \mathcal{I}.
\end{cases}
\]

1Similar to the use of index \( i \in \mathcal{I}_1 \) to denote the decision variables of intersection \( (i, j) \), the convention of using the first cell of an intersection cell-pair to denote the intersection is widely used throughout the paper.
Notice that ordinary, origin and intersection cells require the introduction of a maximization term, creating a multi-objective problem. To avoid this situation, we introduce into the objective function the penalty minimization term \(a \sum_{i \in D} t y_{i,t},\) where \(a\) is a penalty constant, which forces traffic to move outside a given cell as soon as possible, eliminating to a certain extent the unindented vehicle holding-back problem.

B. Additional constraints

Apart from the constraints associated with the CTM model, we need to introduce more constraints regarding the traffic signal operation.

1) Maintaining minimum green time: For the operation of traffic lights, a minimum and maximum green time (GMIN and GMAX respectively) is usually assumed for each phase. To check that the minimum green time constraint is preserved, it must be ensured that for a time window of GMIN + 1 there is no more than one modification in the value of \(w^t_i.\) In mathematical terms this can be expressed as:

\[
\sum_{\tau=t}^{t+GMIN} |w^\tau_i - w^{\tau-1}_i| \leq 1, \quad t = 0, 1, ..., T - GMIN
\]  

(7)

The absolute value operator can be eliminated from the equations by introducing dummy variables \(u^t_i\) which indicate signal changes (green-to-red or red-to-green):

\[
\begin{align*}
&w^t_i - u^{t-1}_i \leq u^t_i, & -w^t_i + u^{t-1}_i \leq u^t_i, \\
&\sum_{\tau=t}^{t+GMIN} u^\tau_i \leq 1, & t = 0, 1, ..., T - GMIN
\end{align*}
\]  

(8)

(9)

2) Maintaining maximum green time: Similar to the minimum green time, the maximum green time constraint is maintained when the same sign is not preserved for more than GMAX time units, which can be expressed as:

\[
\sum_{\tau=t}^{t+GMAX} u^\tau_i \leq GMAX, \quad t = 0, 1, ..., T - GMAX
\]  

(10)

3) Ensuring flow conservation: To ensure flow conservation in our network we must make sure that all entering and initially existing traffic must exit the network. Equivalently, it must be ensured that the network is empty at the end of the considered time horizon. This can be expressed as:

\[
\sum_{i \in A} n_{i,T} = 0
\]  

(11)

C. Initial Condition Constraints

The system state at any time \(t\) is comprised of the cell states, denoted by \(n_{i,t}, i \in A, \forall t,\) as well as the state of the traffic lights \(w^t_k, k \in I, \forall t.\) Computing the next cell state requires only the current cell state in CTM, hence having the initial cell states \(n_{i,0} = n_{i,init}^i\) is sufficient for the evolution of the network. On the contrary, for the correct evolution of the traffic signal decisions we need to consider at least GMAX time units. For example, when GMAX=4 and the four previous decisions at one intersection are ‘1100’, then state ‘0’ can only last for two extra time units, while if the previous decision is ‘0000’, then the next state can only be ‘1’. Hence the decisions at each intersections must be initialized such that \(w^t_k = w^t_{k,init}, k \in I, t = -GMAX, ..., -1.\)

D. Objective

For the MITSC formulation, the minimization of the total travel-time (the cumulative travel time of all vehicles) is considered as the objective. Assuming that the road network is empty both at the start and the end of the simulated time period \(T,\) the total travel delay can be expressed as:

\[
\sum_{i \in D} \sum_{t} t y_{i,t} - \sum_{i \in D} \sum_{t} t D_{i,t}
\]

Notice that the second term is constant and can be eliminated from the objective, while a penalty term must be added to account for the unindented vehicle holding-back problem.

E. MILP formulation

The objective function and constraints for the optimal solution of MITSC can be summarized as:

\[
MITSC : \min \sum_{i \in D} \sum_{t} t y_{i,t} + a \sum_{i \in D} \sum_{t} t D_{i,t}
\]

s.t.: - CTM constraints for all cells,
- constraints (8)-(11),
- initial conditions.

The temporal decomposition of the problem into time-windows of duration \(T_w\) implies that the incoming traffic for that period is non-zero; nonetheless, to accomplish good performance the problem must be optimized for a time-period large enough to allow all the vehicles to exit the network. In formulation MITSC, the decision variables are the traffic light states, \(w^t_i,\) while there are also auxiliary variables \(y_{i,t}, n_{i,t}, q_{i,t}, u^t_i\) that allow the evolution of CTM dynamics and the representation of the GMIN and GMAX constraints.

IV. DISTRIBUTED SOLUTION STRATEGIES

A. Motivation

The centralized solution of the MITSC problem may provide better performance when known, but it has several shortcomings as a solution strategy: (a) it is unsuitable for online execution due to its large-scale nature and NP-hard complexity, (b) it is prone to communication failures, and (c) failure of the central unit results in complete failure of the system.

On the other hand, distributed strategies can be more robust to failures. For example, communication failure between certain intersections may result in partial coordination loss, but intersections can continue working either individually or in subgroups. Additionally, by temporally and spatially decomposing the problem into subproblems of smaller size computationally efficient solutions can be obtained which is essential for online decision making.

Temporal problem decomposition refers to the process of separating the time horizon \(T\) over which MITSC is optimized into smaller time-windows, \(T_w,\) and solving the
problem sequentially over those periods. In fact, when an online strategy is sought, traffic signal plans must be updated every $T_w$ time units. In addition, temporal problem decomposition takes advantage of accurate prediction of traffic demand, which can only be achieved over short time periods (10-15 minutes) [11].

Spatial problem decomposition refers to the process of partitioning the geographical area over which the problem is optimized into small regions. A good policy towards this direction, is to divide the considered area in regions of one intersection, so that each intersection controller decides its own schedule; solutions towards the global optimum can be attained through collaboration with neighboring intersection controllers. It should be emphasized that such a problem need not involve all intersections of a city but rather parts of the city where traffic is interrelated.

In the subsequent sections, we describe the two proposed distributed solution strategies: (a) the local distributed algorithm (LDA), and (b) the neighborhood distributed algorithm (NDA). Note that the two strategies have the same temporal decomposition but different spatial decomposition; hence, in the next sections we consider a specific time-window rather than the whole time horizon and place more emphasis towards the spatial decomposition.

### B. Local distributed algorithm

In LDA we assume that the transportation network under consideration is spatially decomposed into subareas $A_i$ which only involve intersection $i$; each subarea $A_i$ is associated to subproblem $lSP_i$, which adheres to formulation MITSC. We assume that each intersection $i$, has a controller $IC_i$ that is equipped with communication and computation capabilities, while it can also update the intersection signal timing plan according to the derived solution.

$IC_i$ is responsible for solving subproblem $lSP_i$. This can be achieved if complete information about the incoming flows is available to the controller. In case the incoming flows of $lSP_i$ are from the outside of the network, $IC_i$ can directly solve the optimization problem to attain an optimal signal timing plan for the next time-window. Otherwise, the incoming flows of $lSP_i$ emanate from other subproblems, so controller $IC_i$ must wait for incoming flows from neighboring controllers. Let $R_i$ denote the set of controllers from which $IC_i$ receives information, and $S_i$ the set of controllers to which $IC_i$ sends information. Hence, if $R_i \neq \emptyset$, $IC_i$ must wait for all $IC_j \in R_i$ to solve $lSP_j$ and communicate their associated outgoing flows (which become incoming flows for $lSP_i$) before computing the solution to $lSP_i$.

This implies that there must be no node in the network that both sends and receives traffic either explicitly or implicitly from another node, which is true when there are no cycles in the graph formed by the subareas of the transportation network. This is a standard result for deadlock avoidance when allocating resources to concurrent processes in the field of operating systems (e.g. see [12], ch. 6). Such cases are almost true during the morning or evening commute when the overwhelming majority of the traffic is going in or out of the city respectively.

Fig. 1 illustrates an example. The figure shows the CTM model of a 2x2 grid topology with four intersections and one-way arterials. The dashed lines indicate the spatial decomposition of the network into four subareas and hence four subproblems $lSP_i$, $i = 1, \ldots, 4$. Note that $lSP_1$ and $lSP_2$ only have external incoming traffic, while $lSP_3$ and $lSP_4$ have incoming traffic emanating from $lSP_1$ and $lSP_2$. In this topology, intersection controllers $IC_1$ and $IC_2$ can optimize their signal plans based on accurate estimations of their incoming flows for the whole time-window considered and the initial state of $lSP_1$ and $lSP_2$ respectively, at the start of the particular time-period. Once their signal plans are optimized, their outgoing flows can be communicated to $IC_3$ and $IC_4$ (at intersections (0107,0407) and (0208,0307) respectively), so that subproblems $lSP_3$ and $lSP_4$ can be optimized. To validate this property in a general topology, one must form a graph with nodes $A_i$ corresponding to subareas and vertices indicating traffic flows between subareas, and check that there are no cycles in the graph, as shown in Fig. 2(a). LDA is summarized in algorithm 1.

### C. Neighborhood distributed algorithm

In the neighborhood distributed algorithm (NDA) the considered geographical region is divided into subareas $A_i$ as in LDA, but each subproblem $nSP_i$ considers apart from $A_i$, neighboring areas $A_k$ such that $IC_k \in \{R_i \cup S_i\}$. For instance in the 2x2 topology, $nSP_3$ associated with subarea $A_3$ also considers subareas $A_1$ and $A_2$ as shown in Fig. 2(b). Because $nSP_1$ and $nSP_2$ are anti-symmetrical to $nSP_3$ and $nSP_4$, they are omitted from Fig. 2(b). In NDA, decision variables only involve intersection $i$, while signal timing plans associated with neighboring areas $A_k$...
are fixed to values communicated from \( IC_k \) to \( IC_i \). Hence although the formulation of \( nSP_i \) is larger than \( ISP_i \), the number of binary variables remains the same. Intertwining intersections allows for an iterative procedure to take place which progressively improves the solution. This procedure is summarized in Algorithm 2. In Steps 1-2 of the algorithm, each IC computes and communicates to neighboring ICs an initial signal timing plan via LDA. Then, an iterative procedure of two phases is followed. In the first phase (Step 3), \( IC_i, i \in I_1 \), computes and communicates a signal timing plan derived from the \( nSP_i \) formulation. In the second phase (Step 4), ICs exchange information regarding the objective value to decide whether the signal timing plan obtained from this iteration is the best so far. The procedure followed in Steps 4.1-4.6 is similar to the LDA algorithm, except that instead of computing a signal timing plan, each IC computes the part of the objective value associated with output flows that are within its area \( A_i \). Notice that Step 4.5, requires exchange of information between all ICs rather than only neighbor ICs; this can be avoided by considering more advanced neighborhood-based algorithms for the distributed calculation and consensus of the objective value (e.g. see [13]). Step 5 is necessary in order for the ICs to wait for the end of the current time-window before realizing the best signal timing plan (Step 6). It is imperative that Steps 1-4 finish before the end of the current time-window. In case that it takes longer that \( T_w \) to run \( N_I \) iterations, the best signal timing plan by the end of this period can be realized as it is a valid solution.

V. SIMULATION RESULTS

The effectiveness of the proposed distributed strategies was evaluated for the four-intersection topology depicted in Fig. 1. This topology represents a 2-by-2 one-way arterial network with signalized intersections at cell pairs (103,303), (204,403), (107,407) and (208,307), incoming traffic through cells 101, 201, 301 and 401, and outgoing traffic from cells 110, 210, 309 and 409. The free-flow speed is 60km/h and each time-step is equivalent to 6s, resulting in 100m-long cells. The capacity of all cells is \( Q = 5 \) veh./time unit, and the jam density is \( N = 20 \) veh./cell. The ratio of shock-wave speed over free flow speed is \( W = 1 \). The time horizon considered is \( T = 1200 \) time units (2h), with traffic being generated for 600 time units (1h). For the distributed strategies LDA and NDA the time-window taken is \( T_w = 100 \) (10 min). Also, in NDA we used \( N_I = 10 \) iterations for its iterative procedure. In the evaluation, six different problems are considered. Problems \( LC, MC \) and \( HC \) correspond to low, medium, and high constant incoming traffic flows, while \( LR, MR \) and \( HR \) to low, medium, and high random incoming traffic flows respectively.

Fig. 3 depicts the relative percentage deviation from the best objective values among the strategies considered. Strategies “\( C \)” and “\( CTW \)” correspond to the centralized solutions when the problem is optimized globally (no spatial or temporal decomposition) and when temporal decomposition is used respectively. Due to the penalty constant \( a \) imposed in the objective of problem \( MITSC \) and the possibility of vehicle holding-back, the objective value is evaluated directly from the CTM model and according to the global signal timing plan obtained at the end of the current time-window.

\[ \begin{align*}
\text{Algorithm 1 : Local Distributed Algorithm} \\
\text{for each controller } IC_i \text{ do} \\
1. \text{Estimate incoming flows } D_{k,t}, k \in D_{ISP_i}, \forall t. \\
2. \text{Wait for input flows from } IC_j \in R_i. \\
3. \text{Compute solution to problem } ISP_i, \text{i.e. the signal timing plan } w_{k,t}, k \in D_{ISP_i}, \forall t. \\
4. \text{Communicate related output flows } y_{k,t}, k \in D_{ISP_i}, \forall t \text{ to } IC_j \in S_i. \\
5. \text{Wait until the end of the current time-window.} \\
6. \text{Update signal timing plan for the next time-window.} \\
\text{end for}
\end{align*} \]

\[ \begin{align*}
\text{Algorithm 2 : Neighborhood Distributed Algorithm} \\
\text{for each controller } IC_i \text{ do} \\
1. \text{Obtain initial signal timing plan, } w_{k,t}^{i0}, k \in D_{ISP_i}, \text{ using NDA.} \\
2. \text{Communicate related output flows } y_{k,t}, k \in D_{ISP_i}, \forall t \text{ and } \forall t \text{ to all } IC_j \in S_i. \\
3. \text{// Iterative procedure of the } nSP_i \text{ algorithm.} \\
\text{for } l = 1 \text{ to } N_I \text{ do} \\
1. \text{Wait for input from } IC_j \in R_i. \\
2. \text{Solve problem } nSP_i \text{ according to } MITSC \text{ to obtain the signal timing plan } w_{k,t}^{i,l}, k \in D_{ISP_i}, \forall t \text{ of the } l^{th} \text{ iteration.} \\
3. \text{Communicate related output flows } y_{k,t}, k \in D_{ISP_i}, \forall t \text{ to all } IC_j \in S_i. \\
4. \text{// Check if the } l^{th} \text{ plan is the best so far.} \\
1. \text{Wait for input from } IC_j \in R_i. \\
2. \text{Compute parameters } y_{k,t}^{i,l} \text{ and } n_{k,t} \in A_{ISP_i}, \forall t \text{ according to the CTM model and the signal timing plan obtained at the } l^{th} \text{ iteration.} \\
3. \text{Compute the part of the objective value associated with } ISP_i. \\
4. \text{Communicate related output flows } y_{k,t}, k \in D_{ISP_i}, \forall t \text{ to all } IC_j \in S_i. \\
5. \text{Wait for input on the objective value from all } IC_j. \\
6. \text{If the aggregate objective value is the best so far store it as } w_{k,t}^{i,l}. \\
\text{end for} \\
5. \text{Wait until the end of current time-window.} \\
6. \text{Update signal timing plan for the next time-window according to the best plan } w_{k,t}^{i,l}. \\
\text{end for}
\end{align*} \]
maximum execution times. Despite the fact that all strategies considered, while the error lines denote the minimum and execution times over the time-windows of the time-horizon distributed strategies.

Additionally, notice that the variability of the execution times for CTW is significantly larger compared to the two strategies. LDA has an execution time close to 10min in general. One the other hand, LDA and NDA are both suitable for online execution for the considered problems. LDA has an execution time per IC of less than 1min for all problems considered with an average of 5s, while NDA requires on average 1min for execution, with worse case performance about 2min. Additionally, notice that the variability of the execution times for CTW is significantly larger compared to the two distributed strategies.

VI. CONCLUSIONS & FUTURE WORK

This paper introduced two distributed online strategies for the multiple intersection traffic signal control problem. By spatially and temporally decomposing the transportation network, the problem is solved through the iterative solution of small subproblems by intersection controllers and the exchange of local information (outgoing flows, signal timing plans) between neighboring intersections. The choice of modeling the transportation network using the CTM macroscopic model, has allowed a good trade-off between system detail and model complexity facilitating the optimization of the cycle length, the split plans and the offset between intersections. Our results indicate that our proposed distributed strategies, LDA and NDA, produce close to optimal solutions (within 7% and 2% of centralised solutions for the considered topology) and are suitable for online computation.

There are a number of issues that need to be further investigated. First, the case of a topology with cycles in the subarea-graph needs to be addressed and efficiently solved. Second, polynomial complexity algorithms need to be developed for the solution of individual subproblems that provide excellent solution quality and have predictable execution times. This will guarantee the online execution of a distributed strategy for all possible cases that will need to be addressed. Third, to ensure that any developed traffic signal control strategy provides results close to reality, higher-order more accurate macroscopic traffic flow models need to be considered.

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